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NEW THEORETICAL CONSIDERATIONS ON THE ROUGH TURBULENT FLOW PARAMETERS

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ABSTRACT

The field of rough turbulent flow occupies an important place in the practical applications of hydraulic engineer. It is for this reason that the present study is interested in the most important parameters of this flow namely the characteristic length, the average velocity, the Reynolds number, and the hydraulic diameter. To express these parameters, new theoretical considerations are developed based on the combination of Darcy-Weisbach and Nikuradse rational relationships. The implicit form of the equation which governs the characteristic length has been transformed into an explicit power law by a correlation procedure with a very high coefficient of determination. An exact analytical solution in terms of Lambert function was also developed. Thus, the characteristic length can be evaluated explicitly provided the flow rate, the absolute roughness, the channel bed slope and the aspect ratio of the wetted area are given, which is generally the case in practice. The explicit characteristic length equation has been judiciously used to derive the mean flow velocity relationship. This is in the form of that of Manning-Strickler but with slightly different coefficients. The interest of the new velocity model lies in the fact that the resistance coefficient has been determined analytically contrary to the empirical nature of the Manning and Strickler coefficients. The resistance coefficient is explicitly related to absolute roughness and gravity through a physically justified relationship. The last two parameters studied namely the Reynolds number and the hydraulic diameter were deduced from mathematical manipulations and expressed by simple and practical relationships which do not contain the characteristic length

Keywords: Rough turbulent flow, Moody diagram, Manning-Strickler formula, relative roughness, Darcy-Weisbach friction factor, Nikuradse equation, characteristic length.

INTRODUCTION

Referring to the Moody diagram (1944), one can clearly observe that the rough turbulent flow regime occupies a very large space, larger than that of the transition flow regime. It is located to the right of the diagram. In other words, this means that there is a high probability that the rough turbulent flow regime is encountered, more particularly in practice. It is for this reason that special attention should be paid to this flow regime. For this case, one knows better the Manning-Strickler formula, the so-called simply Manning formula that expresses the mean flow velocity in open channels. It is an empirical formula that did not originate from a theory based on physical principles. It is derived from the fitting of observed data and, therefore, there are limits to its use. To work out his relation, Manning calculated the velocity of the flow in a channel obtained from each of the seven empirical formulas known at his time, such as that of Du Buat (1786). Manning calculated the mean velocity derived from each of the formulas for a given value of the channel bed slope S_0 and varying the hydraulic radius R_h from 0.25 m to 30 m. He obtained a series of average values of the velocity and thus came up with a formula that best fitted the data. Therefore, the formula expresses the average velocity V of the flow in a channel as a function of the hydraulic radius and the channel bed slope both located at the right-hand side of the formula. This one is written as (Chow, 1959):

$$V = \frac{1}{n} R_h^{2/3} \sqrt{S_0} \tag{1}$$

Note that the right side of the formula is multiplied by a factor $1/n$ where n is known as the dimensional Manning's roughness factor whose unit is $s/m^{1/3}$. The factor $1/n$ is denoted k which is Strickler's roughness coefficient. Due to the empirical nature of Manning's formula, the coefficient n, or k, is deduced from experimentation and observation. There is no exact method of selecting the value of these coefficients and only a sound experience and judgment could lead to their estimation. For beginning engineers, different measurements would give different values of the coefficients n and k. Manning pointed out that interpolations using empirical correlations that contain dimensional coefficients can only be done between observed data. They cannot be extended or extrapolated for data outside the observation range. It is the values of the slope of the channel and the range of observed data that limit the range of applicability of Manning's formula. According to Falvey (1987), Manning's formula must be applied for $S_0 > 0.0007$ and further, as stated by Christensen (1984), the applicability of this formula is restricted to the following relative roughness range:

$$0.001 \le \varepsilon^* \le 0.01 \tag{2}$$

where $\varepsilon^* = \varepsilon / D_h$, ε is the absolute roughness (m) or the average height of channel surface roughness, and D_h is the hydraulic diameter (m).

According to Moody chart (1944), it can be observed that in the rough turbulent flow domain the relative roughness ε^* varies between 0.0001 and 0.05, meaning that Manning's formula does not apply to an extremely wide zone of the relative roughness. Due to the dimensional coefficient in Manning's formula and all the restrictive considerations mentioned above, Eq. (1) should be discarded in favour of a homogeneous relationship. That amounts to saying that the concern to find a more rational relationship to replace Manning's formula is justified and legitimate. Currently, the most efficient and accurate relationship is undoubtedly that of Darcy-Weisbach (1854) which is also universally applicable.

According to the Darcy-Weisbach formula, the average velocity V of a channel flow or pipe is expressed as follows:

$$V = \sqrt{\frac{2gS_0D_h}{f}} \tag{3}$$

where g is the acceleration due to gravity and f is the so-called Darcy-Weisbach friction factor which is a dimensionless coefficient. No restrictions are known on the applicability of equation (3) unlike Manning's formula. It is valid in the whole domain of turbulent flow, more particularly in the fully rough zone, also called rough turbulent flow zone, which concerns the present study.

Unlike Manning's n or Strickler's k coefficients, f is not estimated empirically. All the coefficients of resistance commonly used today are estimated empirically with the exception of the coefficient f.

Nowadays, the most rational relationship which expresses the friction factor f is that of Colebrook (1939). As stated by Falvey (1987) in his conclusion, "the universal use of Colebrook equation is recommended". He essentially recalls that "most of the major engineering organizations in the world are now using the Colebrook equation to estimate the frictional resistance of open and closed conduit flows. Hager (1985) confirmed this assertion by indicating that in Europe the use of Colebrook's formula is more and more preferred over Manning's formula. Colebrook's formula has the advantage of being applied in ranges of hydraulic parameters well beyond those observed experimentally.

According to Colebrook (1939), the friction factor f is expressed as:

$$f = \left[-2\log\left(\frac{\varepsilon^*}{3.7} + \frac{2.51}{R\sqrt{f}}\right)\right]^{-2} \tag{4}$$

where R is the Reynolds number. As it can be seen, Eq. (4) is implicit towards f which must be computed using a trial-and-error approach. Several research workers have proposed approximate relationships to Eq. (4) that have been reported and discussed in the recent study of Zeghadnia et al. (2019).

For the rough turbulent flow, f is obtained from Eq. (4) after writing $R \to \infty$. The Nikuradse relationship is then reproduced:

$$f = \left[-2\log\left(\frac{\varepsilon^*}{3.7} \right) \right]^{-2} \tag{5}$$

The present study is based on equations (3) and (5) to derive a new relationship of the average velocity of a flow in open channels, applicable in the rough turbulent domain. Given the universality of equations (3) and (5), the derived flow velocity formula will also be a universal equation. Even more, it will be derived the characteristic length of the considered channel or conduit, whether it is the width of the channel or the diameter of the filled or partially filled circular conduit, or even the base width of a trapezoidal channel …etc. The characteristic length can be also the flow depth h. The calculation of this characteristic length will be explicit and will not require any estimation of the flow resistance coefficient. This is no longer the case when equations (3) and (5) are used simultaneously.

CHARACTERISTIC LENGTH

The characteristic length L could be one of the above mentioned non-exhaustive parameters. One of the problems of uniform-flow computation is to evaluate the characteristic length L when the above five other variables are given, namely the discharge Q, the aspect ratio η of the wetted area, the channel bed slope S_0, the absolute roughness ε or Strickler's k roughness coefficient, and the kinematic viscosity ν. If the flow is in the fully rough zone, where effects of viscosity are neglected, the computation can be performed by the use of the continuity equation along with a uniform-flow formula as that of Manning-Strickler expressed by Eq.(1).

The water area A and the wetted perimeter P can be written respectively as:

$$A = L^2 A_1 \tag{6}$$

$$P = L P_1 \tag{7}$$

where the non-dimensional parameters A_1 and $P_1$are respectively the water area and the wetted perimeter when the characteristic length L is equal to unity (one meter). Thus, the hydraulic diameter $D_h = 4A/P$ is as:

$$D_h = 4L \frac{A_1}{P_1} \tag{8}$$

Table 1 groups together some formulae of $A_1$and $P_1$for various channel shapes in accordance with the chosen characteristic length L.

Combining Eqs. (3) and (5) along with the continuity equation $V = Q/A$ results in the following conveyance relationship valid for a channel of any shape, when the flow is in the rough turbulent regime:

$$\frac{Q}{\sqrt{S_0}} = 4\sqrt{2g}\,L^{5/2}\,\frac{A_1^{3/2}}{P_1^{1/2}}\log\left(14.8\frac{LA_1}{\varepsilon P_1}\right) \tag{9}$$

Eq. (9) can be written in dimensionless form by multiplying both the left and the right-hand sides by the following quantity:

$$\delta = \frac{A_1}{P_1^2\sqrt{g}\,\varepsilon^{5/2}} \tag{10}$$

Thus, after some arrangements, one may obtain what follows:

$$\varphi = 4\sqrt{2}\beta^{5/2}\log(14.8\beta) \tag{11}$$

where:

$$\varphi = \frac{\delta Q}{\sqrt{S_0}} = \frac{A_1 Q}{P_1^2\sqrt{gS_0}\,\varepsilon^{5/2}} \tag{12}$$

and β is as:

$$\beta = \frac{LA_1}{\varepsilon P_1} \tag{13}$$

Eq. (11) is the implicit relation that gives the exact value of the characteristic length L. The known parameter is the function φ, what is sought is the β parameter in order to deduce L in accordance with Eq. (13). The values of A_1 and P_1 are given in Table 1 according to the shape of the considered channel.

Taking into account Eq. (8), Eq. (13) can be rewritten as:

$$\beta = \frac{0.25}{\varepsilon / D_h} \tag{13a}$$

Thus, when the relative roughness ε/D_h varies within the wide practical range 0.0001to 0.05 in the rough turbulent zone (Moody, 1944), the corresponding range of β is $5 \le \beta \le 2500$. In this range, deep correlation analysis has shown that Eq. (11) can be formulated as the following power law, with a coefficient of determination R^2 greater than 0.999:

$$\varphi = 8.85\beta^{2.65} \tag{14}$$

Assuming this result, the characteristic length L, defining a linear dimension of a channel of any shape, is easily deduced from equation (14) along with Eqs. (12) and (13). The final result is:

$$L = \left(\frac{Q\varepsilon^{0.15}}{8.85\sqrt{gS_0}} \right)^{1/2.65} \frac{P_1^{\;0.245}}{A_1^{0.623}} \tag{15}$$

Eq. (15) is explicit and does not require knowing the flow resistance coefficient which is implicitly contained in this relationship. Eq. (15) is relevant for all forms of canals and conduits. Observe that the characteristic length L is given as the product of two functions. The first one, which is represented by the quantity in parentheses, has the dimension of a length. The second one is dimensionless and contains both A_1 and P_1 parameters, meaning that it depends exclusively on the aspect ratio as indicated in table 1. Four parameters must be given to calculate L, namely the flow rate Q, the channel bed slope S_0, the absolute roughness ε, and the aspect ratio. Kinematic viscosity ν is not required since Eq. (15) is valid in the rough turbulent flow zone for which the effect of viscosity is negligible.

Eq. (15) can be rewritten in the following reduced form:

$$L = \Lambda\, L^* \tag{16}$$

where:

$$\Lambda = \left(\frac{Q\varepsilon^{0.15}}{8.85\sqrt{gS_0}} \right)^{1/2.65} \tag{17}$$

$$L^* = \frac{P_1^{\;0.245}}{A_1^{0.623}} \tag{18}$$

The condition on the average value of ε for the establishment of the hydraulically rough flow was formulated by Hager (1989) as:

$$\varepsilon \geq 30\,\nu \left[Q(gS_0)^2 \right]^{-0.2} \tag{19}$$

Eq. (15) is valid provided the condition expressed by Eq. (19) is respected. It is worth noting that the deviation between the exact value of the characteristic length given by Eq. (11) and the approximate value computed using Eq. (15) is less than 1% only, as has been confirmed by numerous calculations and that the next numerical examples corroborate. Eq. (15) is interesting insofar as it allows a fast and acceptable calculation of the order of magnitude of the characteristic length.

On the other hand, it is interesting to know that the function Λ has a physical meaning. To do so, let's define in a wide rectangular channel, a square-shaped flow slice of width b and depth $h = b$ such that $\eta = b/h = 1$ (Fig. 1).

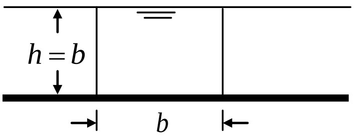

Figure 1: Square-shaped flow slice cut out from a wide rectangular channel

Choosing $L = h$ as the characteristic length, the water area A can be written as:

$$A = bh = h^2(b/h) = h^2A_1$$

Since $b/h = 1$, then one can write:

$$A_1 = 1$$

The wetted perimeter P is as:

$$P = b = h(b/h) = hP_1$$

Since $h = b$, then one can deduce that:

$$P_1 = 1$$

According to Eq. (18), one may write what follows:

$$L^* = \frac{P_1^{0.245}}{A_1^{0.623}} = 1$$

Inserting this result into Eq. (16) yields:

$$L = \Lambda L^* = \Lambda = h$$

This clearly shows that the function Λ corresponds to the depth h of a square-shaped flow slice cut out from a wide rectangular channel.

EXACT ANALYTICAL SOLUTION

On may obtain an exact solution of Eq. (11) without resorting to the explicit approximate Eq. (15). For that, let us express Eq. (11) as follows:

$$\varphi = \frac{4\sqrt{2}}{\ln(10)}\beta^{5/2}\ln(14.8\beta) \qquad (20)$$

which can be written in compact form as:

$$\beta^{5/2}\ln(14.8\beta) = C \qquad (21)$$

where:

$$C = \frac{\varphi \ln(10)}{4\sqrt{2}} \tag{22}$$

The exact analytical solution of the transcendental Eq. (21) can be formulated in terms of the Lambert W function. This solution for $C > 0$ reads then:

$$\beta = \frac{5}{74}\exp\left(\frac{2}{5}\mathbf{W}\left(\frac{2738\sqrt{370}}{25}\ C\right)\right) \tag{23}$$

In which $\mathbf{W}$ is the Lambert Function defined as the inverse function of $z = we^{w}$ i.e. $w = \mathbf{W}_k(z)$ for some integer k. Since the argument of the function is a real number, W-function has two branches namely W_0 and W_{-1}. On account of its importance in mathematics, physics and engineering, the Lambert W function was included in various software such as Maple and Mathematica. As it is a transcendental function, formal solution of the Lambert W-function can be expressed only in endless form. However, a perusal of Eqs. (12) and (22) indicates that the argument x involved in $\mathbf{W}_0(x)$ is very large. For $x \to \infty$, the following three-term asymptotic development holds (Boyd, 1998):

$$\mathbf{W}_0(x) = \ln(x) - \ln\ln(x) + \frac{\ln\ln(x)}{\ln(x)} \tag{24}$$

Using Eq.(24), the computation of the Lambert function is greatly simplified. When substituted in Eq. (23), the exact solution for β and hence for the characteristic length L is worked out from Eq. (13).

EXAMPLE 1

In order to compute the diameter D of a partially filled circular conduit, carrying a uniform flow in a rough turbulent flow zone, the following parameters are given: the discharge Q= 100 *l/s*, the absolute roughness ε = 0.5 *mm*, the channel bed slope S_0 = 0.001, the aspect ratio η = 0.6 corresponding to (Table 1) $\alpha = \cos^{-1}(1-2\eta)$ = 1.772 rd., and the kinematic viscosity ν = 0.000001 m^2/s. Since the unknown parameter is the characteristic length $L = D$, table 1 indicates that, for a partially filled circular conduit, the dimensionless parameters A_1 and P_1 are respectively:

$A_1 = (\alpha - \sin\alpha \cos\alpha)/4 = 0.492$
$P_1 = \alpha = 1.772$

Applying Eq. (15) results in:

$$L = D = \left[\frac{100\times10^{-3}\times\left(0.5\times10^{-3}\right)^{0.15}}{8.85\times\sqrt{9.81\times0.001}}\right]^{1/2.65}\times\frac{1.772^{0.245}}{0.492^{0.623}}\approx 0.513\,m$$

Consequently, the flow depth is:

$$h = \eta\, D = 0.6 \times 0.513 \approx 0.308m$$

Let's check if the flow is in the rough turbulent zone by applying Eq. (16). Thus:

$$\varepsilon \geq 30\,\nu\left[Q(gS_0)^2\right]^{-0.2} = 30\times10^{-6}\times\left[100\times10^{-3}\times(9.81\times0.001)^2\right]^{-0.2}\approx 0.0003\,m = 0.3\,mm$$

The condition is satisfied since ε calculated is less than ε given in the statement of the problem.

Besides the present result, the exact analytical solution of the problem can be used. Eq. (12) gives:

$$\varphi = \frac{A_1 Q}{P_1^2\sqrt{gS_0}\,\varepsilon^{5/2}} = \frac{0.492\times100\times10^{-3}}{1.732^2\times\sqrt{9.81\times0.001}\times(0.5\times10^{-3})^{5/2}} = 28296164.94$$

Thus, Eq. (22) allows writing:

$$C = \frac{\varphi\ln(10)}{4\sqrt{2}} = 11517766.71$$

The Lambert $\mathbf{W}_0(x)$ function can then be calculated from Eq. (24) for the argument x such as:

$$x = \frac{2738\sqrt{370}}{25}\;C = 24264009931$$

leading to the following approximate value of $\mathbf{W}_0(x)$ expressed by Eq. (24):

$$\mathbf{W}_0(x) = \ln(24264009931) - \ln\ln(24264009931) + \frac{\ln\ln(24264009931)}{\ln(24264009931)} = 20.87062034$$

This value is highly accurate to the second decimal place compared with the exact value of $\mathbf{W}_0(x)$ computed by Maple software such as:

$$\mathbf{W}_0(24264009931) = 20.87376682$$

Replacing the approximate value of $\mathbf{W}_0(x)$ in Eq. (23) leads to the value of β as:

$$\beta = \frac{5}{74}\exp\left(\frac{2}{5}\times20.87062034\right) = 285.3227384$$

Finally, the characteristic length $L = D$ is deduced from Eq. (13) as:

$$L = D = \beta \frac{\varepsilon P_1}{A_1} = 285.3227384 \times \frac{0.5\times10^{-3}\times1.772154248}{0.492028357} = 0.51382801\ m$$

Checking the validity of the result

The exact value of the characteristic length $L = D$ can be obtained by solving the implicit equation (11), using an iterative process or using the exact analytical solution when an exact value of $\mathbf{W}_0(x)$ is utilized. The function φ was computed previously as:

$$\varphi = 28296164.9$$

Thus, Eq. (11) is written as:

$$28296164.9 - 4\sqrt{2}\beta^{5/2}\log(14.8\beta) = 0$$

Using an iterative process, the solution of this equation is obtained as:

$$\beta = 285.68206973$$

According to Eq. (13), one may write:

$$L = D = \frac{\beta\varepsilon P_1}{A_1} = \frac{285.68206973\times0.5\times0.001\times1.732}{0.492} = 0.514475\ m$$

This result is the same that one obtains once the exact value of $\mathbf{W}_0(\mathrm{x}) = 20.87376682$ is reported in Eq. (23) leading to $\beta = 285.6820694573$ which in turn gives $L = D = 0.514475\,m$

The deviation between the exact value of D obtained from the iterative process or the analytical solution and that computed explicitly using the proposed method from Eq. (15) is:

$$100\times\left|\frac{0.514475 - 0.51305438}{0.514475}\right| \approx 0.276\%$$

This deviation is only of 0.13% if in the analytical solution the Lambert function $\mathbf{W}_0(x)$ is computed from the approximate expression (24).

EXAMPLE 2

Compute the depth h in a triangular channel for the following data:

$Q = 0.2\ m^3/s$; $\varepsilon = 0.0009\ m$; $S_0 = 0.0005$; $\alpha = 45°$; $\nu = 0.000001\ m^2/s$

For the tilt angle $\alpha = 45°$, one may obtain $m = \text{cotg}\ \alpha = 1$.

Let's check if the flow is in the rough turbulent zone by applying Eq. (16). Thus:

$$\varepsilon \geq 30\,\nu\left[Q(gS_0)^2\right]^{-0.2} = 30\times10^{-6}\times\left[0.2\times(9.81\times0.0005)^2\right]^{-0.2} = 0.00034726\ m \approx 0.35\,mm$$

The condition is satisfied since ε calculated is less than ε given in the statement of the problem.

The only characteristic length for the case of the triangular canal is the depth h. Table 1 indicates that:

$A_1 = m = 1$
$P_1 = 2(1 + m^2)^{1/2} = 2.82842712$

According to Eq. (15), one may write:

$$L = h = \left[\frac{0.2\times0.0009^{0.15}}{8.85\times\sqrt{9.81\times0.0005}}\right]^{1/2.65}\times\frac{2.82842712^{0.245}}{1^{0.623}} = 0.56605196\ m$$

The exact analytical solution of the problem using the Lambert function is as follows:

$$\varphi = \frac{A_1 Q}{P_1^2\sqrt{gS_0}\,\varepsilon^{5/2}} = \frac{1\times0.2}{2.82842712^2\times\sqrt{9.81\times0.0005}\times0.0009^{5/2}} = 14689744.1$$

From Eq. (22), the constant C is computed as:

$$C = \frac{\varphi\ln(10)}{4\sqrt{2}} = 5979363.12$$

The Lambert $\mathbf{W}_0(x)$ function is then calculated from Eq. (24) for the argument x such as:

$$x = \frac{2738\sqrt{370}}{25}\ C = 12596480709$$

The approximate value of $\mathbf{W}_0(x)$ from Eq. (24) is, therefore:

$$\mathbf{W}_0(x) = \ln(12596480709) - \ln\ln(12596480709) + \frac{\ln\ln(12596480709)}{\ln(12596480709)} = 20.2453892$$

This value is to be compared with the exact one computed by Maple software which is:

$$\mathbf{W}_0(12596480709) = 20.24859776\,.$$

Replacing this exact value of $\mathbf{W}_0(x)$ in Eq. (23) leads to the value of β:

$$\beta = \frac{5}{74}\exp\left(\frac{2}{5}\times20.24859776\right) = 222.474374$$

The characteristic length $L = h$ is then deduced from Eq. (13):

$$L = h = \frac{\beta \varepsilon P_1}{A_1} = \frac{222.474374 \times 0.0009 \times 2.82842712}{1} = 0.5663273\ m$$

Note that if the approximate value of $\mathbf{W}_0(x)$ was used from Eq. (24) in the preceding calculation, this would give the characteristic length $L = h = 0.56560092\ m$ which deviates only by 0.128 % from the exact value of h.

Let's check the previous calculation result by determining the exact value of h from the implicit equation (11) by an iterative process. We have:

$$\varphi = 14689744.1$$

So, Eq. (11) is written as:

$$14689744.1 - 4\sqrt{2}\beta^{5/2}\log(14.8\beta) = 0$$

Adopting an iterative scheme, the previous equation gives β as:

$$\beta = 222.474374$$

Which is the same value obtained from the exact analytical solution.

The deviation between the exact value of h and that computed explicitly using the proposed method Eq. (15) is:

$$100 \times \left| \frac{0.5663273 - 0.56605196}{0.5663273} \right| \approx 0.0486\%$$

MEAN FLOW VELOCITY RELATIONSHIP AND RELATED ROUGHNESS COEFFICIENT

Assuming $Q = VA$ and $R_h = LA_1/P_1$, Eq. (15) gives the mean velocity of the flow as the following relationship:

$$V = K R_h^{0.65} \sqrt{S_0} \tag{19}$$

Where K ($m^{0.35}/s$) is the roughness coefficient defined as:

$$K = \frac{8.85\sqrt{g}}{\varepsilon^{0.15}} \tag{20}$$

That is:

$$\frac{K\,\varepsilon^{0.15}}{8.85\sqrt{g}}=1 \tag{21}$$

Eq. (19) constitutes a new formulation of the mean velocity V of a rough turbulent flow. There are no restrictions in its application because it results from a combination of universally accepted rational relationships, unlike the Manning-Strickler formula which has limits of applicability. Note that the exponent of the hydraulic radius in equation (19) is slightly smaller than it is in the Manning-Strickler formula.

Eq. (20) is interesting insofar as the resistance coefficient K can be directly evaluated from the absolute roughness ε alone. As it can be observed, the roughness coefficient K is independent of the hydraulic radius. It depends on both gravity and absolute roughness, which is physically justified.

According to Hager (1987), Strickler's k roughness coefficient is expressed as:

$$\frac{k\,\varepsilon^{1/6}}{8.2\sqrt{g}}=1 \tag{22}$$

Combining Eqs. (21) and (22) results in:

$$\frac{k}{K}=\frac{0.926}{\varepsilon^{1/60}} \tag{23}$$

In the wide practical range 0.1 $mm \leq \varepsilon \leq 50$ mm, the ratio k/K varies between 0.974 and 1.080 according to Eq. (23), thus showing that the variation range is not so wide.

On the other hand, Eq. (19) can be written under the form of Chezy's equation such that:

$$V=C\,R_h^{0.5}\sqrt{S_0} \tag{24}$$

where C is the Chezy's resistance coefficient. Comparing Eqs. (19) and (24) results in:

$$C=K\,R_h^{0.15} \tag{25}$$

Combining Eqs. (20) and (25) yields:

$$C=\frac{8.85\sqrt{g}}{\left(\varepsilon/R_h\right)^{0.15}} \tag{26}$$

Or:

$$\frac{C\left(\varepsilon/R_h\right)^{0.15}}{8.85\sqrt{g}}=1 \tag{26a}$$

This is the relationship linking Chezy's resistance coefficient C to both the relative roughness ε/R_h and the acceleration due to gravity g, in the fully rough turbulent state of the flow.

Comparing Eqs (26a) and (22), one may derive the following C-k relationship:

$$C = 1.08\, k\, \varepsilon^{1/60} R_h^{0.15} \tag{27}$$

REYNOLDS NUMBER

Let R be the Reynolds number characterizing the flow in the rough turbulent domain. Generally, it is well known that R is defined as:

$$R = \frac{4Q}{P\,\nu} \tag{28}$$

According to Eq. (16) for $L = P$, one may write:

$$P = \Lambda\, P^* \tag{29}$$

Thus, Eq. (28) becomes:

$$R = \frac{4Q}{\Lambda\, P^*\, \nu} \tag{30}$$

Eqs. (7) and (29) along with Eq. (16) give what follows:

$$L\, P_1 = \Lambda\, P^* \tag{31}$$

That is:

$$\frac{L}{\Lambda} = L^* = \frac{P^*}{P_1} \tag{32}$$

Whence:

$$P^* = L^* P_1 \tag{33}$$

Inserting Eq. (18) into Eq.(33) results in:

$$P^* = \frac{P_1^{1.245}}{A_1^{0.623}} \tag{34}$$

The real value of the exponents of A_1 and P_1 are in fact equal to 0.62264151 and 1.24528302 respectively. These numbers have been rounded off to their current values, i.e. 0.623 and 1.245. Thus, it is easy to show that Eq. (34) can be written as:

$$P^* = \left(\frac{P_1^2}{A_1} \right)^{0.623} \tag{35}$$

It is useful to note that whatever the chosen characteristic length L, i.e. whether linear dimension characterizing the geometry of the considered channel or flow depth, the computed value of P^* according to Eq. (35) remains unchanged.

Inserting Eq. (35) into Eq. (30) yields:

$$R = \frac{4Q}{\Lambda\, \nu} \left(\frac{A_1}{P_1^2} \right)^{0.623} \tag{36}$$

Eq. (37) is the compact form of the Reynolds number R of a flow in the turbulent rough regime. It can be developed taking into account Eq. (17). After some simplifications and arrangements, the final result is:

$$R = \frac{8.767}{\nu} \left(\frac{Q^{1.65} \sqrt{g S_0}}{\varepsilon^{0.15}} \right)^{1/2.65} \left(\frac{A_1}{P_1^2} \right)^{0.623} \tag{37}$$

Eq. (37) is valid for any shape of channels when the flow is in the rough turbulent zone. According to Eq. (37), the characteristic length L is not required to compute the Reynolds number R which depends on the discharge Q, the channel bed slope S_0, the absolute roughness ε, the aspect ratio η related to A_1 and P_1 as shown in table 1, and the kinematic viscosity ν.

HYDRAULIC DIAMETER

The practical relationship of the hydraulic diameter encompassing a minimum of data can be derived by inserting Eq. (15) into Eq. (8). After some arrangements, one may obtain what follows:

$$D_h = 1.757 \left(\frac{Q\, \varepsilon^{0.15}}{\sqrt{g\, S_0}} \right)^{1/2.65} \left(\frac{A_1}{P_1^2} \right)^{1/2.65} \tag{38}$$

The hydraulic diameter is thus presented as a function of the flow rate Q, the absolute roughness ε, the channel bed slope S_0, and the aspect ratio. Note that the characteristic length is not required as in the case of the Reynolds number R. From Eq. (38) the particular case of the filled circular pipe for which $D_h = D$ can be deduced. Table 1 in the appendix indicates that $A_1 = \pi / 4$ and $P_1 = \pi$. Thus, Eq. (38) becomes:

$$D = 0.676\left(\frac{Q\,\varepsilon^{0.15}}{\sqrt{g\,S_f}}\right)^{1/2.65} \tag{39}$$

In this case, the channel bed slope S_0 has been replaced by the slope S_f of the hydraulic grade line or the hydraulic gradient, also called the friction slope.

CONCLUSION

The study was devoted to the parameters of a flow in the rough turbulent domain insofar as this one occupies an important place in the hydraulic engineering practice. One of these parameters was the characteristic length which defines the linear dimension of any channel shape. It has been shown that this characteristic length is governed by an implicit equation encompassing all the parameters which govern the flow with the exception of viscosity [Eq.(11)]. The implicit character of the equation was removed thanks to a deep correlation analysis process which showed that this relation can be replaced by an explicit power-law with a coefficient of determination greater than 0.999 [Eq.(14)]. Thanks to this, it was shown that the characteristic length is equal to the product of two functions [Eq. (16)]. The first function depends on three parameters, namely the flow rate Q, the absolute roughness ε, and the channel bed slope S_0. The second one depends only on a single parameter which is the aspect ratio of the wetted area, the expressions of which are grouped together in Table 1 according to the chosen characteristic length L. Moreover, an exact analytical solution of the implicit equation was obtained in terms of the Lambert function which simplifies notably the direct solution without resorting to an iterative process computation.

The second parameter of the rough turbulent flow studied is the mean flow velocity V. It has been clearly shown that V can be put in the form of the Manning-Strickler equation with slightly different coefficients [Eq.(19)]. Unlike with the Strickler roughness coefficient k, the resistance coefficient K related to the new velocity model is not empirical. It has been determined analytically and is closely related to the absolute roughness through an explicit physically justified relationship [Eq(20)].

The last two parameters of the rough turbulent flow studied are the Reynolds number R and the hydraulic diameter D_h. Following mathematical manipulations, these two parameters were expressed by very practical relationships which did not contain the characteristic length L [Eqs. (37) and (38)].

Future research will focus on both transition and smooth flow regimes which also occupy an important place in many applications related to laboratory or field testing. The question of how to correct the characteristic length L_R of the rough turbulent flow will have to be resolved if the flow is actually in the transition or smooth domains. It will be necessary to add to Eq. (16) another function that should depend both on the Reynolds number and on the relative roughness when the flow is in the transition domain but will depend only on the Reynolds number when the flow is in the smooth state.

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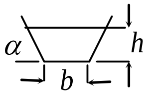
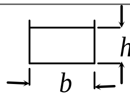
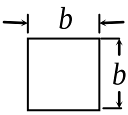
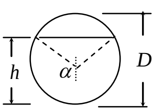
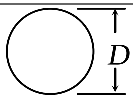
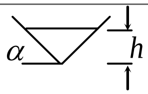
APPENDIX

Table 1: Formulae governing the dimensionless parameters A_1 and P_1 for some channel shapes

Channel shape	Aspect ratio	A	P	L	A_1	P_1
	$\eta = b/h$ $m=\text{cotg}\,\alpha$	$bh+mh^2$	$b+2h(1+m^2)^{1/2}$	b	$\eta^{-1}(1+m\eta^{-1})$	$1+2\eta^{-1}(1+m^2)^{1/2}$
				h	$\eta + m$	$\eta+2(1+m^2)^{1/2}$
	$\eta = b/h$ $m = 0$	bh	$b+2h$	b	η^{-1}	$1+2\eta^{-1}$
				h	η	$\eta+2$
	$\eta = 1$	B^2	$4B$	B	1	4
	$\eta=h/D$ $\alpha=\cos^{-1}(1-2\xi)$	$D^2(\alpha-\sin\alpha\cos\alpha)/4$	αD	D	$(\alpha-\sin\alpha\cos\alpha)/4$	α
				H	$(\alpha-\sin\alpha\cos\alpha)/(4\eta^2)$	α/η
	$\eta = 1$	$\pi D^2/4$	πD	D	$\pi / 4$	π
	$\eta=0$ $m=\text{cotg}\,\alpha$	mh^2	$2h(1+m^2)^{1/2}$	h	m	$2(1+m^2)^{1/2}$