



A DIMENSIONLESS THEORY FOR THE ECONOMIC DESIGN OF PRESSURIZED PIPES

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ABSTRACT

The economic design of pressurized pipes has traditionally been formulated as an optimization problem balancing capital investment against pumping energy expenditure. Although numerous computational techniques have been proposed for determining the optimum pipe diameter, the mathematical structure of the governing equations has received comparatively little attention. The present study revisits the classical problem from an analytical viewpoint by developing a rigorous dimensionless formulation based on the Darcy-Weisbach equation and the Colebrook-White resistance law.

A compound variable combining the Reynolds number and the Darcy-Weisbach friction factor is first introduced, followed by a systematic nondimensionalization of both the hydraulic and economic equations. This analytical procedure transforms the conventional optimization problem into a compact dimensionless framework governed by two reduced variables. It is further demonstrated that the investment and operating cost coefficients naturally combine into a single dimensionless economic parameter representing the relative contribution of energy expenditure with respect to capital investment. Consequently, the optimal design is shown to depend on the balance between these competing mechanisms rather than on their individual dimensional values.

The proposed formulation preserves the full hydraulic rigor of the Colebrook-White equation while considerably simplifying the mathematical structure of the optimization problem. Beyond its immediate application to pipe sizing, the resulting framework provides a clearer physical interpretation of the interaction between hydraulic resistance and economic performance, thereby establishing a unified analytical basis for the economic design of pressurized pipeline systems.

Keywords: Pressurized pipes, Economic optimization, Darcy-Weisbach equation, Colebrook-White equation, Dimensionless analysis, Dimensionless formulation, Hydraulic design, Life-cycle cost.

INTRODUCTION

Pressurized pipeline systems constitute the backbone of modern water conveyance infrastructure and play a fundamental role in municipal water supply, irrigation networks, industrial installations, hydropower developments and long-distance water transfer schemes. Their hydraulic performance directly affects the reliability, efficiency and sustainability of water delivery systems, while their economic performance strongly influences both initial investment and long-term operating costs. As global water demand continues to increase owing to population growth, urban expansion, agricultural intensification and industrial development, the optimization of pressurized pipeline systems has become an increasingly important issue in hydraulic engineering. Simultaneously, the growing concern for energy conservation, environmental protection and sustainable management of water resources has shifted engineering practice from purely hydraulic considerations toward integrated techno-economic design approaches. Unlike many hydraulic structures whose construction costs largely dominate their life-cycle expenditure, pressurized pipes are characterized by a continuous interaction between capital investment and operating expenses. Increasing the pipe diameter generally decreases the mean flow velocity, thereby reducing friction losses, pumping head and energy consumption throughout the operational life of the system. However, this improvement is accompanied by higher material, transportation and installation costs. Conversely, selecting a smaller diameter reduces the initial investment but increases hydraulic resistance and consequently the energy required for pumping. The economically optimal design therefore corresponds to the best compromise between these two competing tendencies and has traditionally been defined as the pipe diameter that minimizes the total life-cycle cost (Shamir and Howard, 1968; Wood and Rayes, 1981; Larock et al., 2000; Walski et al., 2003).

The search for this optimum has attracted continuous attention for several decades because even a slight reduction in total cost may represent substantial financial savings over the service life of large hydraulic systems. This consideration has become even more relevant in recent years owing to increasing electricity prices, stricter environmental regulations and the worldwide commitment to reducing energy consumption and greenhouse gas emissions. Consequently, modern hydraulic design increasingly incorporates economic optimization as an integral component rather than a subsequent verification step. Similar concerns have also emerged in studies devoted to sustainable management of water resources, optimization of hydraulic infrastructures and efficient operation of water distribution systems (Argaz, 2018; Aroua, 2022, 2023; Aroua-Berkat and Aroua, 2022; Goran and Jelisavka, 2016; Khebizi et al., 2023).

The hydraulic behavior of pressurized conduits is governed primarily by friction losses generated along the internal pipe surface. Since the pioneering experimental investigations of Darcy (1854) and the theoretical developments of Weisbach (1845), the Darcy-Weisbach equation has become the universal relationship for evaluating head losses in pressurized pipe flow. Owing to its rigorous physical basis and broad range of applicability, this equation remains the standard formulation adopted in hydraulic engineering regardless of pipe material, flow conditions or system configuration. The

subsequent introduction of the Reynolds number established the physical distinction between laminar and turbulent flow regimes and provided one of the fundamental similarity parameters of fluid mechanics.

A decisive advance was achieved through the work of Colebrook and White (1937, 1939), who established the well-known implicit relationship linking the Darcy-Weisbach friction factor simultaneously to the Reynolds number and the relative roughness of the conduit. The exceptional agreement of this formulation with experimental observations rapidly led to its universal adoption in engineering practice. Shortly afterwards, Moody (1944) transformed the Colebrook-White equation into the celebrated Moody diagram, which remains one of the most widely used engineering tools for estimating turbulent friction losses. During the following decades, numerous explicit approximations were proposed to eliminate the iterative solution required by the Colebrook-White equation, among which the correlations developed by Swamee and Jain (1976), Churchill (1977), Chen (1979) and several subsequent authors are probably the best known. More recently, comprehensive reviews have confirmed that improving friction-factor computation continues to attract considerable research interest because of its direct impact on hydraulic calculations and optimization procedures (Brkić, 2011).

Although the hydraulic formulation of pipe-flow resistance has reached a high degree of maturity, the associated economic optimization problem remains considerably more complex. The total cost of a pressurized pipeline depends simultaneously on hydraulic resistance, flow rate, pipe material, installation conditions, pump characteristics, operating schedule, electricity tariff and economic assumptions adopted during life-cycle analysis. Consequently, the hydraulic and economic components of the problem are intrinsically coupled, making the determination of the optimal diameter substantially more difficult than the solution of the hydraulic equations alone.

The first rigorous attempts to integrate hydraulic performance and economic considerations into a unified design methodology may be traced back to the pioneering work of Shamir and Howard (1968), who introduced optimization concepts into the analysis of water distribution systems. Their contribution established the conceptual basis upon which most subsequent economic design methodologies have been developed. Later investigations by Wood and Rayes (1981) focused on the reliability and numerical behavior of pipe-network algorithms, whereas the comprehensive works of Larock et al. (2000) and Walski et al. (2003) considerably expanded the theoretical and practical aspects of pipeline-system analysis, simulation and optimization. Together, these studies constitute the foundation of contemporary economic design procedures for pressurized pipeline systems.

In parallel with these developments, hydraulic engineering has experienced remarkable progress in analytical modelling, numerical simulation and computational hydraulics. Advanced numerical methods have significantly improved the analysis of pressurized transient flows, hydraulic structures and water-distribution systems (Amara et al., 2016; Amara and Berreksi, 2018; Chaoui et al., 2016; Berrezel et al., 2023; Hariri Asli and Nazari, 2021; Hariri Asli and Hozouri, 2021). Likewise, substantial advances have been achieved in the theoretical analysis of spillways, hydraulic jumps, gradually varied flows

and flow measurement structures through rigorous analytical developments and improved dimensionless formulations (Achour and Amara, 2021, 2023, 2026a, 2026b; Achour et al., 2022a, 2022b, 2025; Amara and Achour, 2020; Houichi and Achour, 2021; Achour and Houichi, 2025; Gouryev et al., 2020; Brakeni et al., 2021; Burlachenko et al., 2022, 2023; Bouriche et al., 2023).

Despite these significant advances, it may be observed that the vast majority of existing investigations have concentrated either on improving computational procedures or on developing more efficient optimization algorithms. Comparatively little attention has been devoted to the mathematical structure of the governing equations themselves. In particular, the possibility that the economic design problem may possess an underlying dimensionless similarity structure has received very limited consideration. This observation provides the primary motivation for the present investigation.

The continuous development of computational hydraulics has profoundly transformed the design philosophy of pressurized water conveyance systems. During the last four decades, increasing computational power has enabled engineers to address optimization problems of unprecedented complexity involving nonlinear objective functions, multiple design variables, operational constraints and stochastic demand scenarios. Consequently, considerable research efforts have focused on improving optimization algorithms rather than revisiting the mathematical formulation of the governing hydraulic equations themselves.

Various optimization techniques have been successfully applied to pipeline and water-distribution systems, including linear and nonlinear programming, dynamic programming, genetic algorithms, simulated annealing, particle swarm optimization and other metaheuristic approaches. These techniques have considerably improved the computational efficiency of engineering design and have expanded the practical applicability of optimization methods to large-scale water distribution systems. Nevertheless, irrespective of the optimization strategy adopted, virtually all existing methodologies continue to employ the Darcy-Weisbach equation together with the Colebrook-White resistance law as the hydraulic model governing head losses.

This situation reflects the remarkable robustness of the Darcy-Weisbach formulation, whose physical validity has never been questioned. Instead, research has mainly concentrated on improving the numerical treatment of the resistance equation or accelerating its evaluation through explicit approximations. Consequently, the mathematical structure of the optimization problem itself has remained essentially unchanged since the pioneering studies devoted to the economic design of pressurized pipe systems.

A similar evolution can be observed in several branches of hydraulic engineering. Significant advances have been achieved through increasingly sophisticated numerical models capable of accurately reproducing complex hydraulic phenomena. Finite-volume, finite-element and computational fluid dynamics formulations have considerably improved the simulation of hydraulic jumps, spillways, side weirs, transient flows, sediment transport and hydraulic structures (Amara et al., 2016; Amara and Berreksi,

2018; Amara et al., 2019; Bentalha and Habi, 2017, 2019; Bouriche et al., 2023; Ghouini et al., 2024). Likewise, numerical investigations devoted to stilling basins, submerged outlets and energy dissipation structures have substantially contributed to improving hydraulic design methodologies (Gouryev et al., 2020; Brakeni et al., 2021; Burlachenko et al., 2022, 2023; Khanov et al., 2026).

Parallel developments have also taken place in hydrological modelling and water-resources engineering. Advanced numerical tools have been successfully employed for rainfall-runoff simulation, flood forecasting, watershed analysis, groundwater assessment, reservoir management and climate-change impact studies (Abdi and Meddi, 2015; Atallah et al., 2024; Fernando et al., 2021; Kherde et al., 2024; Cherki, 2019; Athmani et al., 2025; Ben Said et al., 2024; Hafnaoui et al., 2022, 2023). More recently, machine-learning techniques and artificial intelligence have emerged as promising alternatives for predicting hydraulic and hydrological variables, including sediment transport, scour development, reservoir temperature, groundwater fluctuations and hydraulic-jump characteristics (Ansari et al., 2024; Belouz et al., 2025; Dhimmar and Popawala, 2026; Fatemi and Molavi, 2025; Chabokpour and Azamathulla, 2025).

Although these studies have considerably expanded the computational capabilities available to hydraulic engineers, they share a common characteristic. In nearly all cases, the principal objective consists of improving prediction accuracy, computational efficiency or numerical robustness while preserving the conventional mathematical description of the governing hydraulic processes. Comparatively little attention has been devoted to investigating whether the governing equations themselves may admit alternative analytical formulations capable of providing deeper theoretical insight into the underlying physical mechanisms.

During the last decade, however, an increasing number of analytical studies have demonstrated that suitable variable transformations and nondimensional formulations may reveal mathematical properties that remain hidden in conventional dimensional equations. This tendency is particularly evident in recent investigations devoted to open-channel hydraulics. Rigorous analytical formulations have been developed for gradually varied flows in rectangular and triangular channels (Achour and Amara, 2021; Achour et al., 2022), hydraulic-jump control by broad-crested and thin-crested sills (Achour and Amara, 2023; Achour et al., 2022a, 2022b), critical-flow computation (Achour and Amara, 2026), normal-depth determination (Achour and Amara, 2026a, 2026b) and discharge measurement using modified hydraulic structures (Achour et al., 2025). In each case, the introduction of carefully selected dimensionless variables has led to analytical formulations that are simultaneously more compact and more physically transparent than classical derivations.

Comparable developments have also been reported for spillway hydraulics, submerged overflow structures and energy dissipation systems. The analytical investigations of Houichi and Achour (2021), Achour and Houichi (2019, 2025), Gouryev et al. (2020), Brakeni et al. (2021), Burlachenko et al. (2022, 2023) and Benmalek et al. (2022) collectively illustrate the growing role of dimensionless reasoning in the theoretical interpretation of hydraulic phenomena. Rather than introducing empirical corrections,

these studies seek to identify reduced variables capable of describing complex hydraulic behavior within more general analytical frameworks.

An analogous trend may also be identified in water-distribution engineering. Recent investigations have addressed leakage analysis, water-age modelling, decision-support systems, transient flow simulation and hydraulic-network management through increasingly rigorous mathematical formulations (Chaoui et al., 2016; Berrezel et al., 2023; Hariri Asli and Nazari, 2021; Hariri Asli and Hozouri, 2021). These contributions confirm that analytical reformulation often provides insights that extend beyond numerical computation alone and may considerably improve both theoretical understanding and engineering interpretation.

From a broader scientific perspective, the history of fluid mechanics repeatedly demonstrates that major theoretical advances frequently arise from the identification of appropriate similarity variables rather than from increasingly sophisticated numerical procedures. The Reynolds number, the Froude number, the Euler number and many other dimensionless groups have fundamentally transformed hydraulic engineering because they revealed relationships that remained obscured in the original dimensional equations. Their importance lies not merely in reducing the number of variables, but in exposing the physical balance between the dominant mechanisms governing fluid motion.

This observation naturally raises the question of whether the economic design of pressurized pipes might also possess an underlying similarity structure that has remained largely unnoticed. Although investment costs, operating costs and hydraulic resistance are traditionally treated as distinct components of the optimization problem, their mathematical coupling suggests the possible existence of reduced variables capable of expressing these interactions in a more compact and physically meaningful manner. Surprisingly, this perspective appears to have received very limited attention in the available literature.

Despite the remarkable developments achieved in computational hydraulics and engineering optimization, an important observation may be drawn from the foregoing discussion. Most existing investigations have sought either to improve numerical solution procedures or to increase the computational efficiency of existing formulations. Comparatively fewer studies have examined whether the governing equations themselves admit an alternative analytical representation capable of providing a more fundamental understanding of the economic design problem.

This distinction is of particular importance. Numerical optimization and analytical reformulation pursue different scientific objectives. The former seeks efficient procedures for obtaining the optimum solution, whereas the latter attempts to reveal the mathematical structure governing the problem itself. Throughout the history of engineering science, these two approaches have frequently complemented one another, with major theoretical advances often resulting from the identification of appropriate transformations rather than from the development of increasingly sophisticated computational algorithms.

From this perspective, the economic design of pressurized pipes presents several features that deserve further consideration. The total cost function simultaneously involves hydraulic resistance, flow conditions, pipe geometry and economic parameters, all of which interact through nonlinear relationships. In conventional formulations, these variables appear in their dimensional form and their mutual interactions remain embedded within the governing equations. Consequently, although the optimization problem can be solved numerically, its intrinsic mathematical organization is not immediately apparent.

Dimensionless analysis has historically provided an effective means of revealing hidden relationships in complex engineering problems. By reducing the number of independent variables and identifying appropriate similarity parameters, nondimensional formulations frequently expose governing mechanisms that remain difficult to recognize within conventional dimensional equations. The success of this methodology in fluid mechanics, heat transfer and structural analysis strongly suggests that similar benefits may also be expected for hydraulic-economic optimization problems.

Interestingly, recent analytical developments in hydraulic engineering have demonstrated that suitable variable transformations may considerably simplify the mathematical description of several classical flow problems. Gradually varied flows, hydraulic jumps, critical-flow conditions, spillway hydraulics, submerged overflow structures and discharge-measurement devices have all benefited from formulations based upon carefully selected reduced variables that simultaneously preserve physical rigor and improve mathematical compactness (Achour and Amara, 2021, 2023, 2026; Achour et al., 2022, 2025, 2026; Achour and Houichi, 2025; Houichi and Achour, 2021; Gouryev et al., 2020; Brakeni et al., 2021; Burlachenko et al., 2022, 2023). These contributions clearly illustrate that the introduction of appropriate nondimensional variables may constitute a genuine theoretical advance rather than a simple mathematical convenience.

Nevertheless, such an approach has not yet been systematically extended to the economic design of pressurized pipes governed by the Colebrook-White resistance law. Although the governing hydraulic equations are well established and the economic objective function has long been recognized, both components continue to be treated largely independently during the analytical development of the optimization problem. Consequently, the possibility that both formulations might be unified within a common dimensionless framework has received little attention.

The present investigation originates from this observation. Instead of seeking an alternative approximation of the Colebrook-White equation or introducing another optimization algorithm, the analysis revisits the mathematical formulation of the classical problem itself. The objective is to identify whether suitable transformations may reveal a simpler analytical representation while preserving the exact physical content of the governing equations. Particular attention is therefore devoted to the introduction of reduced variables capable of simultaneously describing hydraulic resistance and economic optimization within a unified mathematical framework.

Starting from the classical Darcy-Weisbach equation, the Colebrook-White resistance law and a conventional life-cycle cost formulation, the present analysis develops a systematic sequence of analytical transformations leading to a fully dimensionless representation of the optimization problem. The proposed formulation progressively eliminates unnecessary dimensional dependencies and ultimately expresses both the hydraulic constraint and the optimality condition in terms of only two reduced variables. An important outcome of this development is the emergence of a unique dimensionless parameter naturally combining the investment and operating cost coefficients into a single quantity governing the relative importance of both economic contributions.

This result provides a different perspective on the classical optimization problem. Rather than interpreting the optimum diameter as the outcome of balancing several independent dimensional parameters, the analysis suggests that the economic behavior of the system can be characterized through a reduced similarity space in which hydraulic and economic effects become intrinsically coupled. Such an interpretation not only leads to a more compact mathematical formulation but also facilitates the physical understanding of the optimization process by emphasizing the relative interaction between the dominant governing mechanisms.

It is important to emphasize that the proposed formulation does not modify the underlying hydraulic physics. The Darcy-Weisbach equation and the Colebrook-White resistance law remain unchanged throughout the analysis. Likewise, the economic objective function retains its conventional interpretation based upon the combined contribution of capital investment and pumping energy. The originality of the present work therefore resides exclusively in the analytical reformulation of the governing equations through appropriate variable transformations and nondimensionalization, thereby revealing mathematical properties that remain hidden within their conventional dimensional representation.

The analytical developments presented hereafter aim to demonstrate that the classical economic design problem of pressurized pipes may be reformulated within a rigorous dimensionless framework governed by a reduced set of similarity variables. Such a formulation is expected to provide a clearer physical interpretation of the optimization process while preserving the full rigor of the governing hydraulic equations, thereby establishing a unified analytical basis for the economic design of pressurized pipeline systems.

DIMENSIONLESS ANALYTICAL FORMULATION

The objective is to determine the economically optimal pipe diameter D for a prescribed discharge Q , while fully accounting for the Colebrook-White resistance law.

The total cost is written as follows:

$$C_T = C_P + C_E \quad (1)$$

where:

C_P is the pipe cost,

and

C_E is the energy cost.

Following the classical practice adopted in hydraulic optimization, the pipe capital cost is represented by a power-law relationship of the form expressed below (Shamir and Howard, 1968; Walski et al., 2003):

$$C_P = a L D^m \quad (2)$$

where a has units of cost per m^{m+1} , L is the pipe length, D is the pipe diameter, and m is a cost exponent.

The exponent m generally ranges between 1 and 2, depending on the pipe material and the adopted cost model. Typical values are approximately $m \approx 1.2$ for certain thermoplastic pipes, whereas values in the range $m \approx 1.5 - 2$ are commonly reported for metallic pipes. Consequently, the coefficient a is not an empirical fitting parameter; rather, it is determined by regression analysis of actual cost data provided by pipe manufacturers, utility companies, or engineering cost databases. It therefore reflects the capital investment associated with the selected pipe material, manufacturing process, and installation conditions.

The operating cost is directly related to the hydraulic power required to overcome the head loss. Accordingly, the annual pumping cost may be written as indicated below; in other words, the energy cost, coming from the hydraulic power, is written as follows (Wood and Rayes, 1981; Larock et al., 2000):

$$C_E = b Q h_f \quad (3)$$

where b has units of cost.s/ m^4 , Q is the discharge, and h_f is the head loss. The coefficient b incorporates the fluid density, gravitational acceleration, pump efficiency, electricity price, operating period, and, when appropriate, the economic discount factor.

Note that the coefficients a and b are not empirical fitting parameters. They are determined directly from the adopted economic model. The coefficient a characterizes the capital cost of the pipe and depends on the pipe material, manufacturing process, installation conditions, and commercial pricing model. The coefficient b represents the life-cycle cost associated with hydraulic energy losses and is obtained from the electricity price, pumping efficiency, operating period, and, when appropriate, the economic discount factor. Consequently, both coefficients possess a clear physical and economic interpretation and are specified before the optimization process.

Using Darcy-Weisbach for a pressurized pipe, the following can be written:

$$h_f = \frac{8 f L Q^2}{g \pi^2 D^5} \quad (4)$$

Combining Eqs. (1) to (4) yields the following:

$$C_T = a L D^m + \frac{8 b f L Q^3}{g \pi^2 D^5} \quad (5)$$

The Colebrook-White equation is given as follows:

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{\varepsilon}{3.7 D} + \frac{2.51}{R \sqrt{f}} \right) \quad (6)$$

where f is the Darcy-Weisbach friction factor, ε is the absolute roughness, and R is the Reynolds number expressed for pressurized pipe as follows:

$$R = \frac{4 Q}{\pi D \nu} \quad (7)$$

where V is the mean flow velocity, and ν is the kinetic viscosity of the flowing liquid.

Let's introduce the following natural compound variable:

$$X = R \sqrt{f} \quad (8)$$

From Eqs. (7) and (8), the following can be written:

$$\sqrt{f} = \frac{\pi D \nu X}{4 Q} \quad (9)$$

Therefore, the following can be deduced:

$$f = \left(\frac{\pi D \nu X}{4 Q} \right)^2 \quad (10)$$

Substitution of Eq. (10) into Eq. (5) yields what follows:

$$C_T = a L D^m + \frac{b \nu^2 L Q X^2}{2 g D^3} \quad (11)$$

This is the first major simplification: the friction factor no longer appears explicitly.

Now; let's define the following normalized diameter:

$$U = \frac{D}{\varepsilon} \quad (12)$$

and

$$\eta = \frac{X}{U} \quad (13)$$

Thus, one may write the following:

$$D = \varepsilon U \quad (14)$$

and

$$X = \eta U \quad (15)$$

Substituting Eqs. (14) and (15) into Eq. (11) results in the following:

$$C_T = a L \varepsilon^m U^m + \frac{b \nu^2 L Q \eta^2}{2 g \varepsilon^3 U} \quad (16)$$

Eq. (16) can be written in the following dimensionless form:

$$\Pi(\eta, U) = U^m + \Omega \frac{\eta^2}{U} \quad (17)$$

where:

$$\Pi(\eta, U) = \frac{C_T}{a L \varepsilon^m} \quad (18)$$

and

$$\Omega = \frac{b \nu^2 Q}{2 g a \varepsilon^{m+3}} \quad (19)$$

A particularly noteworthy feature of the proposed formulation is that the economic optimization is ultimately governed not by the individual values of the coefficients a and b , but rather by their relative contribution through the single dimensionless parameter Ω . Indeed, after nondimensionalization, the investment cost coefficient and the energy cost coefficient no longer appear independently; they are combined into the unique dimensionless group expressed by Eq. (19).

This result is of considerable theoretical significance, as it reveals that the optimal pipe design is controlled not by the absolute magnitude of the economic parameters, but by the balance between operating costs and capital investment. Consequently, Ω may be

interpreted as a dimensionless economic design parameter that characterizes the relative importance of energy expenditure with respect to construction cost. In this sense, Ω plays a role analogous to that of classical dimensionless numbers in fluid mechanics, such as the Reynolds number, by governing the interplay between two competing physical and economic mechanisms. This interpretation provides a new conceptual framework for understanding the economic optimization of pressurized pipe systems and may constitute the foundation for a generalized dimensionless theory of hydraulic economic design.

On the other hand, substituting Eqs. (14) and (15) into Eq. (9) yields the following:

$$\sqrt{f} = \frac{\pi \varepsilon \nu \eta U^2}{4Q} \quad (20)$$

Let's define the following dimensionless parameter:

$$S = \frac{4Q}{\pi \nu \varepsilon} \quad (21)$$

Thus, Eq. (20) reduces to the following compact form:

$$\sqrt{f} = \frac{\eta U^2}{S} \quad (22)$$

Or

$$\frac{1}{\sqrt{f}} = \frac{S}{\eta U^2} \quad (22a)$$

Substituting Eqs. (14) and (21) into Eq. (7) yields the following:

$$R = \frac{S}{U} \quad (23)$$

Therefore, the product of both Eqs. (22) and (23) results in the what follows:

$$R \sqrt{f} = \eta U \quad (24)$$

Substituting Eqs. (14), (22a) and (24) into Eq. (6) results in the following:

$$\frac{S}{\eta U^2} = -2 \log_{10} \left(\frac{1}{3.7U} + \frac{2.51}{\eta U} \right) \quad (25)$$

Eq. (25) can be written in the following compact form:

$$\frac{S}{\eta U^2} = 2 \log_{10} \left[\frac{U}{K(\eta)} \right] \quad (26)$$

where:

$$K(\eta) = \frac{1}{3.7} + \frac{2.51}{\eta} \quad (27)$$

Let's define the following function:

$$F(\eta, U) = \frac{S}{\eta U^2} - 2 \log_{10} \left[\frac{U}{K(\eta)} \right] \quad (28)$$

The hydraulic constraint is therefore the following:

$$F(\eta, U) = 0 \quad (29)$$

By implicit differentiation, the following can be written:

$$\frac{\partial F}{\partial \eta} + \frac{\partial F}{\partial U} \frac{dU}{d\eta} = 0 \quad (30)$$

Hence,

$$\frac{dU}{d\eta} = - \frac{F_\eta}{F_U} \quad (31)$$

The two partial derivatives are expressed as follows:

$$F_U = - \frac{2S}{\eta U^3} - \frac{2}{U \ln(10)} \quad (32)$$

$$F_\eta = - \frac{S}{\eta^2 U^2} - \frac{5.02}{\eta^2 K(\eta) \ln(10)} \quad (33)$$

Substituting Eqs. (32) and (33) into Eq. (31) yields the following:

$$\frac{dU}{d\eta} = - \frac{\frac{S}{\eta^2 U^2} + \frac{5.02}{\eta^2 K(\eta) \ln(10)}}{\frac{2S}{\eta U^3} + \frac{2}{U \ln(10)}} \quad (34)$$

On the other hand, the economic optimum is obtained from the following:

$$\frac{d\Pi}{d\eta} = 0 \quad (35)$$

From Eq. (17), one may obtain the following:

$$\frac{d\Pi}{d\eta} = \left(mU^{m-1} - \Omega \frac{\eta^2}{U^2} \right) \frac{dU}{d\eta} + 2\Omega \frac{\eta}{U} \quad (36)$$

Thus, according to Eq. (35), the final optimality condition is as follows:

$$\left(mU^{m-1} - \Omega \frac{\eta^2}{U^2} \right) \frac{dU}{d\eta} + 2\Omega \frac{\eta}{U} = 0 \quad (37)$$

The design problem is therefore reduced to solving the first two equations below [Eqs. (26) and (37) along with Eq. (34)]:

$$\frac{S}{\eta U^2} = 2 \log_{10} \left[\frac{U}{K(\eta)} \right] \quad (26)$$

$$\left(mU^{m-1} - \Omega \frac{\eta^2}{U^2} \right) \frac{dU}{d\eta} + 2\Omega \frac{\eta}{U} = 0 \quad (37)$$

$$\frac{dU}{d\eta} = - \frac{\frac{S}{\eta^2 U^2} + \frac{5.02}{\eta^2 K(\eta) \ln(10)}}{\frac{2S}{\eta U^3} + \frac{2}{U \ln(10)}} \quad (34)$$

Once U and η are known, the design diameter is derived from Eq. (14) recalled as follows:

$$D = \varepsilon U \quad (14)$$

In addition, both the variable X and the friction factor are immediately worked out from Eqs. (15) and (10), respectively.

The economic design problem is therefore governed by two coupled equations. Eq. (26) represents the transformed Colebrook-White equation and constitutes the hydraulic constraint of the optimization problem. Eq. (37) is obtained from the stationarity condition of the dimensionless objective function and therefore represents the optimality condition. The simultaneous solution of Eqs. (26) and (37) yields the optimal values of the reduced variables U and η , from which the optimal pipe diameter, friction factor, Reynolds number, and all remaining hydraulic quantities are subsequently determined.

CONCLUSION

A rigorous analytical reformulation of the classical economic design problem for pressurized pipes has been presented. Starting from the conventional life-cycle cost function together with the Darcy-Weisbach equation and the Colebrook-White resistance law, the governing equations were transformed into a fully dimensionless framework through an appropriate sequence of analytical variable transformations.

The proposed formulation demonstrates that the optimization problem can be expressed in terms of only two reduced variables while preserving the exact physical content of the original governing equations. An important outcome of the analysis is the elimination of the explicit dependence on the Darcy-Weisbach friction factor within the objective function, resulting in a substantially simplified mathematical representation of the optimization problem.

A second significant contribution is the identification of a unique dimensionless economic parameter combining the investment and operating cost coefficients into a single governing quantity. This result shows that the optimal pipe diameter is controlled not by the absolute magnitude of the economic parameters taken separately but by their relative contribution to the total life-cycle cost. Such an interpretation provides a clearer understanding of the interaction between hydraulic resistance and economic optimization.

Unlike many previous investigations, the present work does not seek to improve numerical optimization algorithms or to develop new explicit approximations of the Colebrook-White equation. Instead, it revisits the mathematical formulation of the classical problem itself and demonstrates that its governing equations possess a compact dimensionless representation. This reformulation considerably simplifies the theoretical analysis while maintaining complete consistency with the established hydraulic laws.

Beyond the immediate application to economic pipe sizing, the proposed framework opens several promising perspectives. The dimensionless formulation may facilitate the construction of generalized design charts, sensitivity analyses, parametric investigations and scaling relationships applicable to a broad range of hydraulic and economic conditions. It may also provide a convenient basis for extending the methodology to pipe-network optimization, variable roughness conditions and more comprehensive life-cycle economic analyses.

The present study therefore suggests that the economic design of pressurized pipes can be interpreted within a unified dimensionless analytical framework, thereby providing both a more compact mathematical description and a clearer physical interpretation of the optimization process.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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