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AN ACCESSIBLE POTENTIAL ENERGY BUDGET (APEB) FOR DENSITY CURRENTS IN STRATIFIED DAM RESERVOIRS

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ABSTRACT

Density currents in stratified dam reservoirs play a central role in sediment transport, water quality, and reservoir operation, yet existing models primarily describe how such currents evolve once established rather than whether they can persist as coherent flows. Classical gravity-current formulations rely on local force balances, geometric intrusion criteria, or empirical entrainment closures, and do not explicitly quantify the energetic feasibility of sustaining a density current in a stratified environment.

In this study, we introduce an Accessible Potential Energy Budget (APEB) framework that explicitly links reservoir stratification to the mechanical energy available to drive and maintain density currents. The word "budget" is used in a strict physical and energetic sense, accounting of sources and sinks of mechanical energy derived from buoyancy. Thus, the term "Budget" used in this study should be understood as "Balance, Accounting, or Quantification". Starting from classical layer-averaged formulations for mass, buoyancy dilution, and momentum, we identify the neutral depth as a stratification constraint and define a new quantity: the accessible buoyancy work per unit mass, obtained by integrating the ambient density profile along the vertical adjustment path of the inflow. This quantity converts a measured density stratification into an energetic scale directly comparable to dissipation processes.

By multiplying the accessible buoyancy work by the inflow mass flux, we obtain a buoyancy-derived power input that represents the maximum mechanical energy available from stratification. We then introduce an explicit decomposition of the dissipation demand, including (1) internal turbulent mixing and entrainment power, (2) bed-friction losses, and (3) interfacial drag with the ambient fluid. The resulting APEB persistence criterion states that a coherent density current can exist only if accessible buoyancy power exceeds the total dissipation demand.

A worked numerical example using real reservoir temperature-depth data demonstrates how the framework is applied in practice. The example shows how the shape of the ambient density profile strongly controls the accessible energy through a dimensionless stratification factor, explaining why currents with identical inflow density and discharge may persist in some reservoirs but collapse rapidly in others.

The proposed framework extends classical density-current theory by introducing an explicit energetic gatekeeper for current formation and persistence. It unifies underflows and interflows within a single energy-based perspective and provides a physically interpretable criterion that can be evaluated directly from field data, offering new predictive capability for reservoir management and environmental hydraulics.

Keywords: Density currents, Stratified dam reservoirs, Buoyancy-driven flows, Accessible potential energy, Energy budget, Neutral depth, Entrainment and mixing.

INTRODUCTION

Density currents are gravity-driven flows that arise when a fluid of one density moves within another fluid of different density under the action of gravity (Benjamin, 1968; Turner, 1973; Chow and Teo, 2023). In dam reservoirs, density currents are ubiquitous and occur whenever incoming river water differs in density from the ambient reservoir water due to temperature contrasts, suspended sediment concentration (Meguenni and Remini, 2008; Remini, 2010), or dissolved constituents (Akiyama and Stefan, 1984; Fan and Morris, 1992; Morris and Fan, 1998). Once formed, these currents may plunge beneath the surface and propagate along the reservoir bottom (underflows), intrude at intermediate depths (interflows), or spread along the surface (overflows), depending on the relative density of inflow and ambient water (Simpson, 1997; Imberger and Patterson, 1981; Rueda et al., 2007). Because dam reservoirs are typically stratified for much of the year, density currents play a fundamental role in governing sediment transport, water quality, thermal structure, and reservoir operational performance (Wells and Sherman, 2001; Fischer et al., 1979; Baines, 2013).

The study of density currents has a long history in fluid mechanics, geophysical flows, and hydraulic engineering (Benjamin, 1968; Turner, 1973; Simpson, 1997). Early foundational work established that density differences under gravity can generate self-driven currents that propagate independently of externally imposed pressure gradients (Benjamin, 1968; Turner, 1973). Subsequent laboratory and field studies demonstrated that such currents are characterized by strong shear, turbulent mixing, entrainment of ambient fluid, and progressive dilution as they propagate (Ellison and Turner, 1959; Simpson, 1997; Parker et al., 1986). In reservoirs, these processes control whether a density current remains coherent over long distances or rapidly collapses into diffuse mixing with the ambient water column, with direct implications for sediment deposition near dams, clogging of intakes, hypolimnetic oxygen depletion, and the redistribution of heat and dissolved substances (Akiyama and Stefan, 1984; Morris and Fan, 1998; Wells and Sherman, 2001; Rueda et al., 2007).

Relevant studies on density current, associated with dam siltation, are those conducted by Professor Remini, adapted to the arid and semi-arid Algerian context (Remini an Ramin, 2003; Remini and Bensafia, 2016; Remini, 2022). Prof. Remini has developed a coherent body of work at the interface of density-current hydraulics and reservoir siltation management, with a particular emphasis on arid and semi-arid dam reservoirs where short, intense floods generate sediment-laden inflows that plunge and propagate as turbidity/density currents, delivering fine sediment directly to the deep reservoir and dam vicinity (Remini, 1997; Remini and Maazouz, 2018; Remini and Ouidir, 2017).

A central theme across these papers is that density currents constitute a dominant mechanism of siltation in many Algerian reservoirs: sediment-laden flood inflows form a dense layer that travels along the reservoir bottom toward the dam, where it decelerates and deposits large volumes of fine material. This process both accelerates capacity loss and controls the spatial distribution of deposits (near-bed, thalweg-aligned accumulation), explaining why some reservoirs can become heavily silted even when average annual inflows appear moderate (Remini and Maazouz, 2018; Remini, 2019).

A second major contribution in Remini's work is the operational use of density currents as a management tool through withdrawal/venting/flushing, i.e., withdrawing the turbidity current through low-level outlets (Remini, 2017). His Larhyss case study on the Erraguene reservoir synthesizes more than a half-century of operational experience and documents how timely withdrawal of density currents can export a substantial fraction of incoming fine sediment, thereby reduce net accumulation and extend reservoir life (Remini and Ouidir, 2017). The work explicitly connects the hydrodynamic timing of the turbidity current arrival to operational decisions and proposes performance indicators, e.g., long-term evacuated sediment volumes and withdrawal efficiency.

Remini's Larhyss study of Foum El Gherza Dam develops the same conceptual chain but highlights the *opposite* outcome when operational or infrastructural conditions are not favourable: density currents may still form and travel significant distances, yet if outlets are not actuated appropriately, or are limited in configuration, the current stagnates near the dam, settles rapidly, and produces severe siltation. In that paper, density current dynamics are treated as the direct driver of reservoir infilling, and the discussion links observed bathymetric evolution and deposit accumulation to the repeated occurrence of density-current events during floods (Remini and Maazouz, 2018).

In parallel to dam-specific case studies, Remini also contributed a broader, synthesis-style Larhyss paper that frames density currents as a general natural phenomenon in arid environments, while still devoting substantial attention to reservoir turbidity currents and their consequences for siltation. This work is valuable because it provides interpretive guidance on why density currents may (1) fully reach the dam, (2) collapse prematurely due to dilution and geometry, or (3) behave intermittently depending on flood hydrograph and sediment concentration, offering a conceptual bridge between field observations and operational outcomes (Remini, 2019a; Remini, 2019b; Bougamouza et al., 2020).

Beyond Larhyss, Remini and co-authors also produced earlier technical and journal contributions cantered on the Ighil Emda Dam, which is widely cited in his later Larhyss work as a reference case for the feasibility of sediment evacuation by density-current withdrawal. These works discuss the reservoir siltation problem and the principle of exploiting the turbidity current pathway through appropriate outlet operations (Kettab et al., 1995; Remini et al., 1996a; Remini et al., 1996b).

Finally, Remini's broader siltation-focused contributions, including synthesis and management-oriented writings, repeatedly highlight that ignoring density currents leads to underestimation of deep-reservoir sediment delivery and can result in ineffective mitigation strategies. In this view, the density current is not merely a hydraulic curiosity but a practical governing mechanism: it controls where sediment deposits, whether it can be vented, and ultimately whether the reservoir's long-term storage and outlet functionality can be preserved (Remini, 1997).

Research on density currents has traditionally sought to answer three broad questions. First, under what conditions does a density current form when an inflow enters a stratified environment? Second, once formed, how does the current evolve in terms of thickness, velocity, density, and entrainment rate? Third, what controls the ultimate fate of the current, whether it persists as a coherent gravity-driven layer or collapses into background mixing?

Classical studies have focused primarily on the second question, describing how density currents evolve assuming that they already exist and remain dynamically active (Ellison and Turner, 1959; Simpson, 1997; Parker et al., 1986). Predictive tools developed in this context typically describe the downstream evolution of velocity, thickness, or densimetric Froude number, often under quasi-steady assumptions and depth-averaged formulations (Benjamin, 1968; Fukushima et al., 1985; Parker et al., 1986; Baines, 2013). While these models have proven extremely valuable, they do not explicitly address a more fundamental and practically important issue: whether the stratification of the receiving water body provides sufficient mechanical energy to sustain the current against turbulent mixing and frictional losses, particularly in strongly stratified reservoirs (Imberger and Patterson, 1981; Wells and Sherman, 2001; Rueda et al., 2007).

In dam reservoir applications, this unresolved question is critical. Reservoirs with apparently similar inflow conditions may exhibit very different density-current behaviour solely due to differences in ambient stratification. Some inflows generate persistent underflows that reach the dam, while others with comparable density and discharge collapse shortly after plunging. Understanding and predicting this behaviour remains a central challenge in reservoir hydraulics.

The study of density currents has relied on a combination of laboratory experiments, field observations, and mathematical modelling (Turner, 1973; Simpson, 1997; Baines, 2013). Laboratory experiments, often conducted in flumes or tanks, have played a crucial role in isolating fundamental mechanisms such as entrainment, head formation, mixing efficiency, and scaling laws (Ellison and Turner, 1959; Britter and Linden, 1980; Simpson, 1997; Huppert and Simpson, 1980). These experiments have led to widely used empirical relationships for entrainment coefficients and constraints based on the densimetric Froude number, which remain central to many predictive and layer-averaged density-current models (Benjamin, 1968; Turner, 1986; Parker et al., 1986; Baines, 2013).

Field studies in lakes, reservoirs, estuaries, and oceans have complemented laboratory work by revealing the influence of real stratification, irregular geometry, and unsteady inflow conditions on density-current behaviour (Imberger and Patterson, 1981; Johnson et al., 1989; Alavian et al., 1992; Wells and Sherman, 2001). Observations have demonstrated that the ambient density structure can strongly constrain the vertical adjustment, intrusion depth, and spreading pathways of density currents, introducing behaviours not easily reproduced in laboratory settings, such as multi-layer intrusions, intermittent collapse, and strong sensitivity to seasonal stratification (Farmer and Armi, 1986; Rueda et al., 2007; Cenedese et al., 2014; Nash et al., 2012).

Mathematically, density currents are most commonly described using layer-averaged or depth-integrated formulations, originally developed to reduce the fully three-dimensional governing equations to a tractable form while retaining the dominant physical mechanisms (Benjamin, 1968; Turner, 1973; Simpson, 1997). These reduced-order models express conservation of mass, momentum, and buoyancy anomaly for an idealized current layer and have been widely applied to gravity currents, turbidity currents, and reservoir underflows (Ellison and Turner, 1959; Parker et al., 1986; Fukushima et al., 1985; Alavian et al., 1992).

While such formulations successfully capture the essential physics of entrainment, head propagation, and downstream evolution, turbulence production, mixing, and entrainment are typically parameterized through empirical coefficients rather than treated explicitly as energetic processes (Turner, 1986; Simpson, 1997; Wells and Sherman, 2001). As a result, classical layer-averaged models describe how density currents evolve once established but do not directly assess whether the available buoyancy-derived energy is sufficient to sustain the current against turbulent dissipation and frictional losses, particularly in strongly stratified environments (Imberger and Patterson, 1981; Baines, 2013).

More recently, advances in stratified-flow theory have emphasized the role of available potential energy, buoyancy fluxes, and irreversible mixing in determining flow behaviour and efficiency (Winters et al., 1995; Tailleux, 2009; Scotti and White, 2014). In related geophysical contexts, such as oceanic overflows, dense boundary currents, and atmospheric stratified flows, energy-based perspectives have proven highly effective in linking stratification structure to flow persistence, mixing efficiency, and pathway selection (Baringer and Price, 1997; Ivey et al., 2008; Ferrari and Wunsch, 2009). However, despite their success in oceanic and atmospheric applications, these energetic approaches have not yet been systematically applied to reservoir density currents in a form that directly exploits measured field stratification data to assess current persistence and dissipation feasibility (Imberger and Patterson, 1981; Wells and Sherman, 2001).

Over the past decades, extensive work has been conducted on gravity currents and density-driven flows. Early theoretical foundations were laid by Benjamin (1968) and Turner (1973), who clarified the role of buoyancy and stratification in driving motion. Experimental studies by Ellison and Turner (1959) and later by Simpson (1997) established the central importance of entrainment and turbulent mixing. Parker et al. (1986) and Fukushima et al. (1985) developed layer-averaged models for turbidity currents, emphasizing sediment-laden underflows relevant to reservoirs and deep-sea environments.

In reservoir-specific contexts, studies by Fan and Morris (1992), Morris and Fan (1998), and Akiyama and Stefan (1984) examined plunging inflows and stratification-controlled intrusion depths. Field observations by Imberger and Patterson (1981), Wells and Sherman (2001), and Rueda et al. (2007) demonstrated the sensitivity of inflow pathways to ambient density profiles. More recent numerical and experimental investigations have addressed internal waves, interflow stability, and mixing efficiency in stratified reservoirs (Baines, 2013; Özgökmen et al., 2014).

Despite this vast literature, most existing formulations implicitly assume that once a density current forms, it has sufficient energy to persist. The energetic feasibility of maintaining entrainment, turbulence, and drag over long distances is rarely examined explicitly, particularly in relation to measured density stratification profiles.

The present study addresses this gap by introducing a new energy-based perspective for reservoir density currents. Rather than focusing solely on force balances or geometric intrusion criteria, the study explicitly quantifies how much mechanical energy is made accessible by the ambient stratification and compares this energy to the power required to sustain entrainment, mixing, and frictional losses.

In our framework, a *budget* refers to a formal accounting of competing physical contributions (sources and sinks) governing a conserved or convertible quantity, herein mechanical energy derived from buoyancy; it means energy balance, accounting, or allocation of physical quantities. By framing the problem in this way, density-current persistence is treated as an energetic feasibility question rather than a purely kinematic or force-balance problem.

The methodology combines classical layer-averaged descriptions with a new stratification-integrated measure of accessible buoyancy energy, computed directly from measured ambient density profiles. This approach makes it possible to assess, using real reservoir data, whether an inflow possesses sufficient buoyancy-derived energy to overcome dissipation demands and remain coherent.

The objectives of this study are threefold. First, to formalize an Accessible Potential Energy Budget (APEB) that translates dam reservoir stratification into an explicit energetic resource for density currents. Second, to identify and quantify the principal dissipation mechanisms, internal mixing, bed friction, and interfacial drag, in a consistent energetic framework. Third, to derive a persistence criterion stating that a coherent density current can exist only if the accessible buoyancy power exceeds the total dissipation demand.

The originality of the study lies not in redefining classical governing equations, but in introducing a physically interpretable energetic gatekeeper that determines whether a density current can form and persist at all. By linking neutral depth, stratification shape, and dissipation demand within a single framework, the proposed approach extends classical density-current theory and provides new predictive capability for stratified dam reservoirs.

What is a dam reservoir density current?

A density current in a dam reservoir is a gravity-driven flow that forms when incoming water has a different density than the receiving reservoir water (Fig. 1). The density difference can be caused by: (1) Temperature, i.e., cold river inflow into warm reservoir, or warm into cold, (2) Dissolved solids/salinity, and/or (3) Suspended sediment (turbidity).

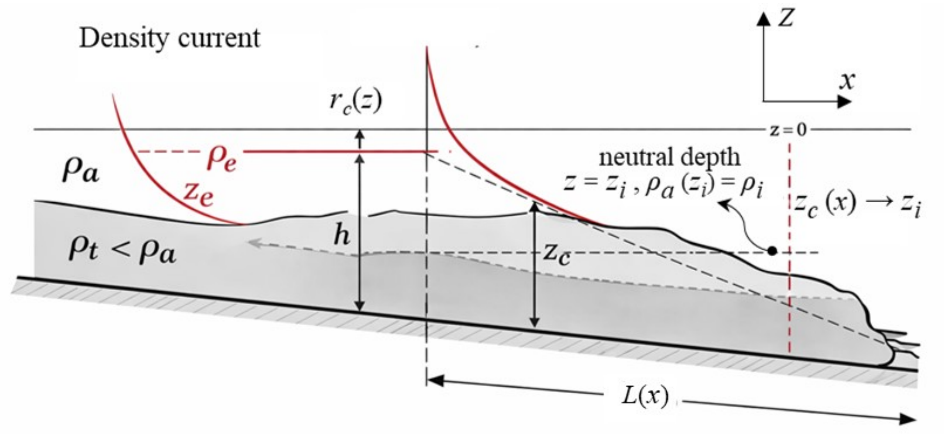

Figure 1: Schematic of a turbidity (density) current in a reservoir showing entrainment, deposition, and head-body structure (Authors' own schematic representation)

Fig. 1 presents a conceptual representation of a density current propagating within a stratified reservoir. An inflow of density ρe enters the reservoir at an effective entry depth ze and encounters an ambient water column characterized by a vertically varying density profile $\rho a(z)$. Owing to the density contrast between the inflow and the ambient water, the inflow plunges beneath the surface and forms a gravity-driven density current that propagates downslope along the reservoir bed. As the current entrains ambient fluid, its density and thickness evolve downstream, and the current progressively adjusts toward its neutral buoyancy depth. The figure highlights the geometric and kinematic variables used to describe the current, including its thickness, lateral extent, and centerline depth, which form the basis of the governing equations and energy-based analysis developed in this study.

Buoyancy framework

It is worth noting that "Buoyancy" is the force per unit mass (or per unit volume) that arises when a fluid parcel is embedded in a fluid of different density and subjected to gravity. It results from the imbalance between the parcel's weight and the weight of the ambient fluid it displaces. In a gravitational field *g*, buoyancy per unit mass is defined as follows:

$$b(z) = g\frac{\rho_a(z) - \rho}{\rho_0}$$

where: ρ is the density of the fluid parcel, e.g. inflow or density current, $\rho a(z)$ is the ambient density at depth z, g is gravitational acceleration, $b > 0$ means upward (positively buoyant), $b < 0$ means downward (negatively buoyant). The above expression is exact under the Boussinesq approximation used in our study.

From a physical point view, and at its most fundamental level, buoyancy measures the tendency of a fluid parcel to move vertically because its density differs from that of its surroundings. Be aware that it is not a velocity, not an energy, and not a pressure; it is a force density (or acceleration) created by gravity acting on density differences. Buoyancy is therefore the root cause of vertical motion in stratified fluids, and the origin of potential energy conversion in density-driven flows.

Moreover, in our framework, buoyancy has a specific and central role:

(1) Buoyancy as the driver of density currents; a density current exists because:

$$\rho_i \neq \rho_a(z)$$

This density mismatch creates buoyancy forces that: push the inflow downward (underflow shown in Fig. 2), pull it upward (overflow, Fig. 2), or guide it toward an intermediate equilibrium (interflow, Fig. 2). Without buoyancy, no density current can exist.

(2) Buoyancy as a source of mechanical energy; In our study, buoyancy is not treated only as a force but as an energy source. When a dense inflow descends through a stratified water column, buoyancy forces do work, gravitational potential energy is released; this energy can be converted into kinetic energy of the current, turbulence, or entrainment and mixing.

This motivates our definition of the buoyancy work per unit mass, which will be detailed in the next sections:

$$\psi = \int g\frac{\rho_i - \rho_a(z)}{\rho_0}$$

Thus, buoyancy is the mechanism by which stratification stores and releases mechanical energy. It is the physical link between stratification, motion, and energy conversion in density currents.

In our framework, it is elevated from a local force to a quantifiable energy source that governs whether a current can exist, persist, and overcome dissipation.

Parameters definition

Below is the definition of all parameters shown in Fig. 2

(1) Coordinates and geometry

x

is the **l**ongitudinal coordinate measured in the downstream direction along the reservoir thalweg (Fig. 1);

z

Is the vertical coordinate, positive upward, with $z = 0$ denoting the free surface;

$L(x)$

Is the length of the current.

z_e

is the effective inflow entry depth, defined as the depth at which the inflow jet first interacts dynamically with the reservoir water column;

$h(x)$

Is the local thickness of the density current, measured vertically from the bed to the upper interface of the current at position x;

$b(x)$

Is the lateral (cross-stream) width of the density current at position x, used to define the cross-sectional area of the current in the y-direction;

$z_c(x)$

is the centreline depth of the density current, typically defined as the depth of maximum velocity or the centroid of momentum and buoyancy.

(2) Density quantities

$\rho_a(z)$

is the ambient reservoir density as a function of depth, representing the background stratification due to temperature, salinity, or dissolved solids; it is sometimes written as:

$a(z)$

in shorthand notation.

ρ_e

is the density of the inflowing water at the point of entry into the reservoir;

ρ_t

is the mean density of the density current at a given downstream location, after entrainment and mixing with ambient water.

$\rho_t < \rho_a(z)$

Indicates that, locally, the density current remains lighter than the surrounding ambient water at that depth, consistent with buoyancy-controlled motion toward neutral equilibrium.

(2) Auxiliary functions

$r_c(z)$

is the vertical distribution function associated with the ambient density (or reduced density anomaly), used to describe how buoyancy forcing varies with depth in the stratified reservoir.

How the density current is produced

When water enters a reservoir, it meets an ambient density field $\rho a(z)$ that often varies with depth (stratification). The inflow density ρi may be: (1) Greater than nearby ambient density, so it tends to sink (underflow / plunging current), (2) Less, so it tends to rise/spread at top (overflow), (3) Intermediate, so it intrudes at a level where it matches ambient density (interflow).

Role in reservoirs

Density currents are “conveyors” that redistribute sediment, temperature, oxygen, and nutrients. They can: (1) deliver sediment to deep zones (capacity loss, intake clogging risk), (2) transport cold/low-oxygen water to outlets (water quality impacts), (3) create or erode stratification, affecting the whole reservoir’s seasonal behaviour.

A reservoir density current persists as a coherent gravity-driven layer only if the inflow’s accessible potential energy flux Φ exceeds the power required to sustain turbulence, entrain ambient fluid, and overcome frictional/form drag. This theory is designed to unify: which layer the inflow chooses (over/inter/underflow) (Fig. 2), and whether the current persists (long runout) or collapses (rapid mixing).

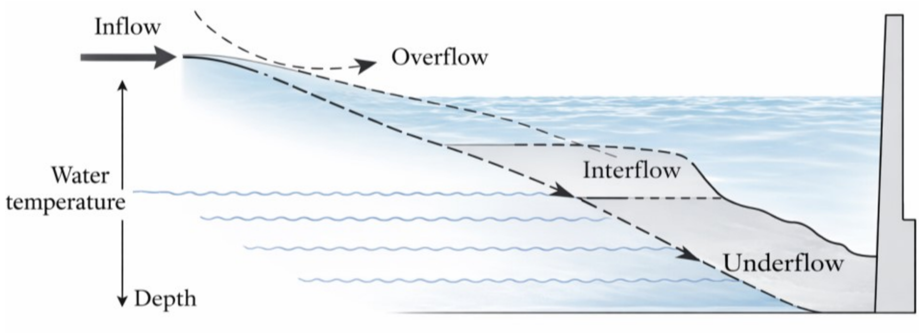

Figure 2: Conceptual representation of stratification-controlled inflow pathways in a dam reservoir (Author's own schematic representation)

Fig. 2 is a conceptual schematic illustrating the fate of an inflow entering a stratified reservoir. Depending on the density of the inflow relative to the ambient reservoir density profile, the inflow may form an overflow that remains near the surface, an interflow that intrudes horizontally at its level of neutral buoyancy, or an underflow that plunges and propagates along the reservoir bed. Blue wavy lines schematically represent ambient stratification in the reservoir (isopycnal or isothermal layers) and are not intended to depict surface waves or turbulence. Arrows indicate the qualitative flow pathways associated with each regime. The schematic is not drawn to scale and is intended solely to illustrate the governing physical mechanisms.

Think of the inflow as arriving with:

a density anomaly relative to the reservoir profile, momentum (discharge/velocity); the reservoir offering: a density stratification profile (how density changes with depth), a geometry (slope, width), and an outlet/withdrawal pattern.

Coordinate system and variables

Let x be along the reservoir thalweg (down-reservoir direction). Let z be vertical upward, with $z = 0$ at free surface and $z = -H(x)$ at bed.

Ambient density profile

Let $\rho a(z)$ be the ambient density profile known from CTD profile, or piecewise continuous.

Inflow properties at entry section x = 0

The inflow properties at entry section $x = 0$, are the following: discharge Q_0, inflow density ρi, inflow momentum scale (velocity) U_0, entry depth ze (often near surface).

Density current (layer-averaged) variables

Figs. 3 presents an original schematic defining the geometric variables used to describe a streamwise-evolving density current in a stratified reservoir. Fig. 4 illustrates a cross-sectional area of the layer at position x.

Let's define the following (Fig. 3): layer thickness $h(x)$, layer length $L(x)$ at position x, layer mean velocity $u(x)$, layer mean density $\rho(x)$ (evolves due to entrainment/mixing), layer cross-sectional area (Fig. 4) $A(x) = b(x) \times h(x)$, and layer discharge $q(x) = u(x) \times A(x) = u(x) \times b(\mathrm{x}) \times h(x)$.

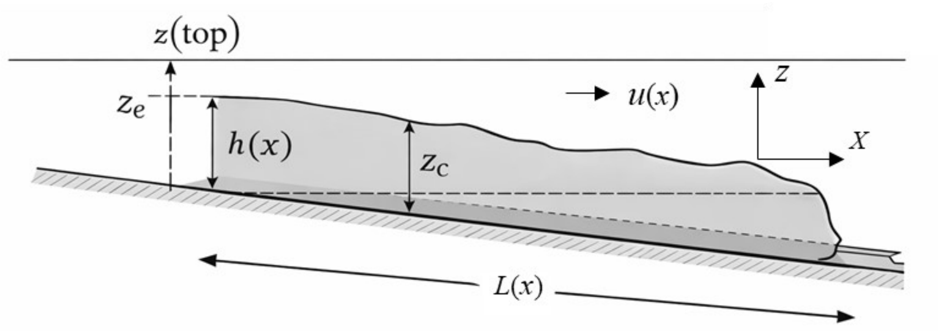

Figure 3: Geometric definition of a streamwise-evolving density current in a stratified dam reservoir (Author's own schematic representation)

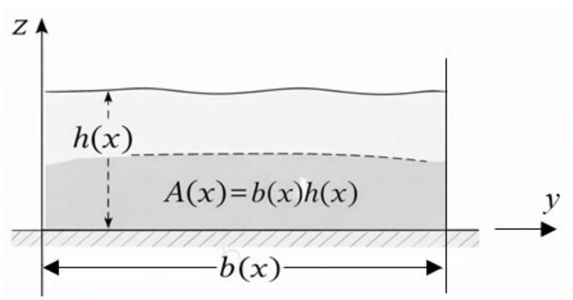

Figure 4: Cross-sectional area of the layer at location *x*

THEORY: CLASSICAL FOUNDATIONS AND NOVEL ACCESSIBLE POTENTIAL ENERGY BUDGET (APEB) DEVELOPMENT

This section is organized to separate classical, literature-established components from the new contributions introduced here. The goal is to keep the governing physics familiar while making the novelty explicit and traceable step-by-step.

Problem statement and modelling objective

As defined previously, we consider an inflow of density

$$\rho_i$$

entering a stratified reservoir characterized by an ambient density profile

$$\rho_a(z)$$

Depending on ρi relative to $\rho a(z)$, the inflow may plunge and propagate as a density current. The objective is twofold: (1) describe the evolution of a density current using a reduced-order (layer-averaged) model; and (2) introduce a new predictive criterion for whether a coherent density current will persist or collapse into mixing.

Items (1) is classical, while item (2) is the authors' novel contribution.

Classical foundation: geometry, variables, and stratification

We adopt a 2D streamwise coordinate x and vertical coordinate z (positive upward), with free surface at $z = 0$. A density current is represented as a layer of thickness $h(x)$ and width $b(x)$ (Fig. 3), with cross-sectional area written as follows:

$$A(x) = b(x)h(x) \tag{1}$$

The layer-averaged velocity is $u(x)$, and the layer discharge is expressed as follows:

$$q(x) = u(x)A(x) = u(x)b(x)h(x) \tag{2}$$

Define the (layer-averaged) density anomaly relative to the ambient at the current's depth as follows:

$$\Delta\rho(x) = \rho_t(x) - \rho_a\left[z_c(x)\right] \tag{3}$$

Under the Boussinesq approximation, the reduced gravity is as follows:

$$g'(x) = g\frac{\Delta\rho(x)}{\rho_0} \tag{4}$$

with ρ_0 is a reference density (≈1000 kg/m^3), and g is gravity.

It is worth noting that Eqs. (1) to (4) are standard in layer-averaged gravity-current theory and reservoir density-current modelling.

Classical foundation: mass conservation with entrainment

Ambient water is entrained into the current at an entrainment velocity *w*e. The downstream increase in layer discharge results solely from entrainment of ambient fluid across the upper interface; under steady conditions, volume conservation therefore yields the following:

$$\frac{d}{dx}(q) = w_e b \tag{5}$$

Eq. (5) translated the fact that the downstream rate of increase of the current discharge equals the rate at which ambient water is entrained across the interface. If *w*e = 0, there is no entrainment and the discharge remains constant downstream. On the other side, if *w*e > 0, the entrainment occurs and the discharge increases monotonically downstream. The equation directly expresses that entrainment dilutes and thickens the current by adding volume, regardless of whether velocity increases or decreases.

A common closure assumes entrainment scales with mean velocity as follows:

$$w_e = Eu \tag{6}$$

where *E* is an entrainment coefficient to be calibrated.

Substituting Eqs. (2) and (6) into Eq. (5) yields the following:

$$\frac{d}{dx}(u\,h\,b) = E\,u\,b \tag{7}$$

The structure of Eqs. (5) to (7) is classical; the novelty does not lie in using entrainment, but in how entrainment is coupled to stratification through the new energy criterion introduced later.

Classical foundation: buoyancy dilution (derivation of the anomaly decay)

Assuming the entrained fluid has density

$$\rho_a(z_c)$$

and neglecting other sources/sinks of density anomaly, e.g., settling, heating, the advected anomaly flux is conserved as follows:

$$\frac{d}{dx}(q\,\Delta\rho) = 0 \tag{8}$$

If the derivative of a quantity is zero, the quantity is then constant, allowing writing the following:

$$q(x)\Delta\rho(x) = \text{constant} \tag{9}$$

Evaluate the constant using the upstream/entry condition at $x = 0$, as follows:

$$q(0)\Delta\rho(0) = q_0\,\Delta\rho_0 \tag{10}$$

So, for any x, the following can be written:

$$q(x)\Delta\rho(x) = q_0\,\Delta\rho_0 \tag{11}$$

Rearranging Eq. (11) yields what follows:

$$\Delta\rho(x) = \frac{q_0}{q(x)}\Delta\rho_0 \tag{12}$$

So, entrainment increases q, which dilutes $\Delta\rho$ and weakens g' according to Eq. (4).

From a physical meaning point view, as entrainment increases the discharge $q(x)$ downstream, the same "initial anomaly content"

$$q_0\,\Delta\rho_0$$

is spread over a larger flow rate, so the anomaly magnitude must decrease:
if $q(x)$ doubles, $\Delta\rho(x)$ halves.

Eq. (12) relies on the assumption that anomaly changes only by dilution. It breaks if you include any additional processes that change density anomaly besides entrainment, for example: sediment settling (reduces anomaly for turbidity currents), heating/cooling, salinity diffusion, entrainment from layers not equal to

$$\rho_a(z_c)$$

strongly varying with depth without proper averaging.

In addition, this "dilution by entrainment" result is standard for entraining gravity currents.

Classical foundation: streamwise momentum balance

The streamwise momentum balance is obtained by vertically integrating the Reynolds-averaged momentum equation over the thickness of the density current, following classical gravity-current theory (Turner, 1973; Simpson, 1997). The resulting equation balances momentum advection with buoyancy forcing and resistive stresses at the bed and at the current-ambient interface. Thus, a compact 1D momentum balance for the layer (steady, along x) can be written as follows:

$$\frac{d}{dx}\left(\rho_0 q u\right) = \rho_0 g' A S_{\text{eff}} - \tau_b b - \tau_i b \tag{13}$$

where S_{eff} is an effective driving slope (bed-following and/or isopycnal-following), and τ_b, τ_i represent bed and interfacial stresses.

The existence of a momentum balance of this form is classical. In this study, the momentum balance is not claimed as a novelty; its role is to provide a physically consistent description of velocity and entrainment levels that appear in the new persistence criterion.

Novel contribution: neutral depth as a stratification constraint

Neutral depth

Neutral depth, indicated as z_i in Fig. 1, is the vertical location in a stratified ambient fluid at which the density of the ambient fluid equals the density of the inflow (or density current), so that the net buoyant force acting on the inflow vanishes. The neutral depth z_i is defined implicitly by the following condition:

$$\rho_a(z_i) = \rho_i \tag{14}$$

This is a stratification constraint: it identifies where buoyant forcing changes sign for the inflow water.

In addition, one may write the following:

If

$$\rho_a(z) < \rho_i \tag{15}$$

near the surface, the inflow plunges downward until it reaches $z = z_i$.

If

$$\rho_a(z) > \rho_i \tag{16}$$

the inflow rises until it reaches $z = z_i$.

Moreover, once near z_i, vertical buoyant acceleration is zero, and the flow preferentially spreads horizontally as an interflow.

Novel step: using neutral depth to quantify accessible potential energy

Classical models recognize zi, but typically do not use it to quantify “how much energy the inflow can actually extract” from the stratification. Here we do exactly that by defining an accessible potential energy per unit mass.

Novel contribution: Accessible Potential Energy Flux Φ

Accessible potential energy per unit mass

Consider a parcel of inflow fluid entering at depth *ze*. Define the accessible buoyancy-work per unit mass associated with moving from *ze* to a target depth *z*∗.

The quantity *z*∗ is a reference vertical position used to evaluate the maximum buoyancy work that an inflowing water mass can release as it adjusts to the ambient stratification. It can be defined as the target depth up to which the inflow (or density current) can move solely under buoyancy forces, starting from its entry depth *ze*. It is not: (1) the actual depth reached at a given downstream location, (2) the instantaneous centreline depth of the current, or a geometric feature of the current itself.
Instead, *z*∗ is a thermodynamic reference depth. In a stratified fluid, buoyancy forces depend on depth because ambient density varies with *z*. To quantify how much potential energy is available, one must specify how far the parcel can move vertically before buoyancy vanishes or changes sign. That "how far" is encoded by *z*∗.
In addition, in reservoir density-current problems, one may state the following: (1) If the inflow is denser than surface waters, it will tend to move downward until it becomes neutrally buoyant; (2) If the inflow is lighter, it will tend to rise; (3) The motion stops when net buoyancy force becomes zero.
Therefore, the physically relevant choice is as follows:

$$z* = z_i \text{, and } \rho_a(z_i) = \rho_i$$

This makes *z*∗ the neutral depth. More precisely, *z*∗ is the depth that bounds the vertical buoyancy adjustment of the inflow. It marks the deepest (or shallowest) position the inflow can reach under buoyancy alone.

Let's consider a parcel of volume *V* at depth *z*. Two vertical forces act on it:

(1) Gravitational weight of the parcel which can be written as follows:

$$F_W = \rho_i V g \text{ , (downward)} \tag{17}$$

(2) Buoyant force from displaced ambient fluid (Archimedes' principle):

$$F_B = \rho_a(z) V g \text{, (upward)} \tag{18}$$

Using sign conventions consistently, i.e., force positive upward, thus, the net vertical force on the parcel is as follows:

$$F_{\text{net}} = F_B - F_W \tag{19}$$

Substituting Eqs. (17) and (18) into Eq. (19) yields what follows:

$$F_{\text{net}}(z) = \rho_a(z)Vg - \rho_i Vg \tag{20}$$

Eq. (20) can be rewritten as follows:

$$F_{\text{net}}(z) = \left[\rho_a(z) - \rho_i\right]Vg \tag{21}$$

This force is

positive upward if $\rho_a(z) > \rho_i$ (parcel is lighter than ambient) and

negative downward if

$\rho_a(z) < \rho_i$ (parcel is heavier than ambient).

Let the parcel move an infinitesimal vertical distance *dz*. A positive *dz* means an upward displacement. Work done by a force over a displacement is written as follows:

$$dW = F_{\text{net}}\, dz \tag{22}$$

Substituting Eq. (21) into Eq. (22) yields the following:

$$dW(z) = \left[\rho_a(z) - \rho_i\right]Vg\,dz \tag{23}$$

This is the work done by buoyancy minus weight on the parcel during a small vertical move.

In our reservoir context, we want the work that can be converted into kinetic energy of the inflow/current. That is the work done by the net buoyancy force (buoyancy relative to ambient). But Eq. (23) is work on a parcel of volume *V*. We want work per unit mass so that it can be applied to a flowing discharge independent of the parcel size.

The mass *m* of the parcel is approximately

$$\rho_0 V \tag{24}$$

under the Boussinesq approximation, since

ρ_i and ρ_a

differ only slightly from a reference ρ_0.

Thus, the following can be written:

$$m \approx \rho_0 V \tag{25}$$

Let's introduce the ratio $d\psi$ = "work/mass" (W/m), representing the specific work, which is the amount of energy transferred (work done) per unit of mass.

The quantity ψ is the accessible buoyancy work per unit mass associated with vertical adjustment of the inflow in a stratified environment. It has units of: $[\psi] = m^2/s^2 = J/kg$. So, ψ is an energy per unit mass.

In addition, classical density-current studies focus on: (1) local force balances, e.g. densimetric Froude number, and (2) geometric intrusion depths. They do not answer the following question: *How much energy does the stratification actually provide to drive or sustain a density current*? The quantity ψ answers exactly that question. Thus, ψ measures the potential energy released (or required) when a water mass of density *ρi* moves from its entry depth *ze* to a target depth *z*∗ in a stratified ambient fluid. Moreover, the following can be written:

If $\psi > 0$, buoyancy releases energy; this fact can drive motion, turbulence, entrainment.
If $\psi = 0$, no buoyancy works available; this is a neutral adjustment.
If $\psi < 0$, buoyancy resists motion; external energy would be required.

In our framework, ψ: Bridges stratification and dynamics as it translates a density profile *ρa*(*z*) into an *energy* scale; Enables a persistence criterion since, when multiplied by inflow mass flux, ψ gives the following accessible potential energy flux:

$$\Phi = \rho_0 Q_0 \psi \tag{26}$$

which can be directly compared to dissipation demands; ψ explains variability beyond Froude scaling, since two inflows with identical *ρi* and velocity may have very different ψ if the stratification differs; and ψ *u*nifies underflows and interflows as the same ψ formulation applies whether the inflow sinks or rises.

Readers should think of ψ as follows: the height of a waterfall in terms of energy, and except the "height" is not geometric but determined by density stratification.

Thus, ψ is a measure of the energetic "fuel" provided by stratification.

Dividing Eq. (23) by Eq. (25), we get thus the work per unit mass as follows:

$$d\psi(z) = \frac{\left[\rho_a(z) - \rho_i\right] V g\, dz}{\rho_0 V} \tag{27}$$

Cancelling *V* in Eq. (27), and rearranging, the following can be written:

$$d\psi(z) = g\frac{\rho_a(z) - \rho_i}{\rho_0} dz \tag{28}$$

This is the incremental buoyancy-work per unit mass for a small displacement *dz*.

Eq. (28) gives the work per unit mass done by the net upward force. Many gravity-current formulations instead define a driving potential that is positive when the parcel moves in the direction favoured by buoyancy. To express the available buoyancy work in a form that is positive when a denser parcel moves downward (or a lighter parcel moves upward), we flip the difference inside the numerator to write the following:

$$d\psi(z) = g\frac{\rho_i - \rho_a(z)}{\rho_0}dz \tag{29}$$

This is equivalent to (28) but corresponds to defining ψ as the work released by buoyancy adjustment rather than work done against it. It is simply a consistent convention: the sign of the integral will correctly reflect whether energy is released or required for the displacement. Now integrate the differential work-per-mass expression over the path from $z = ze$ to a target depth $z = z*$. Thus, the following is written:

$$\psi(z*) - \psi(z_e) = \int_{z_e}^{z*} g\frac{\rho_i - \rho_a(z)}{\rho_0}dz \tag{30}$$

Let's choose the reference $\psi(ze) = 0$, i.e., measure accessible buoyancy-work relative to the entry condition. Then, one may write what follows:

$$\psi(z*) = \int_{z_e}^{z*} g\frac{\rho_i - \rho_a(z)}{\rho_0}dz \tag{31}$$

The integral sums that buoyancy "advantage" over the vertical distance travelled. It is therefore the buoyancy work per unit mass that is potentially convertible into motion (kinetic energy) or dissipated by turbulence.

The integrand:

$$g\frac{\rho_i - \rho_a(z)}{\rho_0} \tag{32}$$

is the reduced gravity a parcel would have relative to the ambient at depth z. Eq. (32) is comparable to Eq. (4) which available in the literature.

It is emphasis to recall that Eq. (31) relies on these assumptions: (1) Boussinesq approximation: $\rho \approx \rho_0$ in inertia terms, but density differences matter in buoyancy; (2) Hydrostatic ambient: ambient pressure field is consistent with $\rho a(z)$; (3) Parcel density treated as ρi during the conceptual displacement from ze to $z*$, i.e., the "accessible" energy is defined from the initial density before dilution; dilution is handled later by the demand term and/or evolution equations.

To compute ψ, the following is needed:

(1) Inflow density ρi derived from temperature/salinity/sediment concentration of inflow.
(2) Ambient density profile $\rho a(z)$ provided from CTD or temperature-conductivity measurements.

(3) Entry depth *ze*, often near the surface but may be deeper.
(4) Target depth $z*$, typically the neutral depth *zi*.

The computation procedure of ψ is as follows:

(1) Determine $z*$ by solving:

$$\rho_a(z*) = \rho_i$$

If such a depth exists, it is the neutral depth.

(2) Set up the integrand [Eq. (32)]. At each depth *z*, compute:

$$f(z) = g\frac{\rho_i - \rho_a(z)}{\rho_0}$$

This represents the local reduced gravity experienced by the inflow.

(3) Integrate vertically computing the following [Eq. (31)]:

$$\psi(z*) = \int_{z_e}^{z*} f(z)dz$$

This can be done numerically using trapezoidal or Simpson's rule.

After calculation following the previous steps, one may observe: Larger ψ indicates more energy available, and more persistent current; Smaller ψ implies limited energy, and rapid collapse or weak intrusion.

The target depth $z*$ defines the limit of buoyancy-driven vertical adjustment of the inflow, while ψ quantifies the buoyancy work per unit mass that is accessible from the ambient stratification over that adjustment. Together, they convert a vertical density profile into an energetic measure that can be used to assess whether a density current can form and persist in a reservoir. This energetic perspective represents a fundamental extension beyond classical geometric or force-based descriptions of density currents.

When considering the natural target depth $z*$ as the neutral depth *zi* , one may writing the following:

$$z* = z_i$$

Thus, Eq. (31) is written as follows:

$$\psi(z_i) = \psi_i = \int_{z_e}^{z_i} g\frac{\rho_i - \rho_a(z)}{\rho_0}dz \tag{31a}$$

This integral is fully determined by measurable stratification $\rho a(z)$ and inflow density ρi

Accessible potential energy flux (new quantity)

Multiplying both sides of Eq. (31a) by inflow mass flux

$\rho_0 Q_0$

yields the following accessible potential energy flux:

$$\Phi = \rho_0 Q_0 \psi_i = \rho_0 Q_0 \int_{z_e}^{z_i} g \frac{\rho_i - \rho_a(z)}{\rho_0} dz \tag{31b}$$

Φ has units of power and represents the maximum buoyancy-driven power available from stratification to support a coherent current.

It is worth noting that the explicit use of a stratification integral to define a *power budget* available to a reservoir density current is the core new construct of this theory.

Numerical example

Below is a worked numerical example of how to compute the neutral depth *zi* and then evaluate the stratification integral ψ [Eq. (31)] using real temperature, depth profile data from an actual reservoir report: The Mark Twain Lake, USA.

1) Real stratification data used (Mark Twain Lake)

From the 2021 Mark Twain Lake Annual Water Quality Report, the "August 15, 1200 Lake Profile" gives the following depths [m] and temperatures *T* [°C] reported in Table 1. This is a real, field-reported thermal stratification snapshot (late-summer stratified conditions).

2) Convert temperature profile to density profile

Because the report profile is freshwater lake/reservoir context, a standard approximation is to treat salinity as negligible and use a pure-water density-temperature polynomial, valid for typical lake temperatures (*T* °C, density Kg/m^3). A commonly used density polynomial (freshwater at ~1 atm) is as follows:

$$\rho_a(z) = \rho_a[T(z)] = 999.842594 + 6.793952\times10^{-2}T - 9.095290\times10^{-3}T^2 + 1.001685\times10^{-4}T^3 - 1.120083\times10^{-6}T^4 + 6.536332\times10^{-9}T^5 \tag{33}$$

Table 1: Real stratification data used (Mark Twain Lake)

No	Depth z (m)	Temperature (°C)	$\rho_a(z)$ kg/m³ Eq. (33)
1	0.3048	28.89	995.981068
2	1.2192	28.89	995.981068
3	2.1336	28.89	995.981068
4	3.3528	28.33	996.14193
5	4.2672	27.78	996.300021
6	5.1816	26.67	996.607783
7	6.4008	25	997.047958
8	7.3152	22.78	997.593557
9	8.2296	21.11	997.970683
10	9.4488	17.78	998.638384
11	10.3632	15	999.101575
12	11.2776	13.89	999.261774
13	12.4968	12.22	999.473961
14	13.4112	11.67	999.536977
15	14.3256	11.11	999.596043

Table 1 indicates in particular the following:

(1) at surface layer

$T \approx 28.9$ °C, $\rho a \approx 995.98$ kg/m^3

(2) at deep layer

$T \approx 11.1$ °C, $\rho a \approx 999.6$ kg/m^3

Thus, the ambient water is denser at depth, which is consistent with thermal stratification. The thermal stratification naturally creates layers where warmer, less dense water stays on top of cooler, denser water, forming stable layers that increase in density with depth due to temperature changes, a key feature of lakes and oceans, a key feature of lakes and oceans.

Fig. 4 shows the vertical distribution of the ambient water density $\rho a(z)$ in the reservoir, derived from the temperature-depth data used in the numerical example. Density increases monotonically with depth, reflecting a thermally stratified water column typical of late-summer conditions. The strong density gradient in the upper few meters corresponds to the epilimnion-metalimnion transition, while the deeper region exhibits a more gradual increase toward near-uniform hypolimnetic density. The lake bed is indicated for reference. This stratification profile directly controls the buoyancy forces acting on an inflow or density current and determines the location of the neutral depth z_i, defined by the condition $\rho a(z_i) = \rho_i$. Consequently, the area between the curve $\rho a(z)$ and the inflow density ρi (when plotted in density-depth space) represents the buoyancy potential available to drive vertical adjustment and horizontal propagation of density currents in the reservoir.

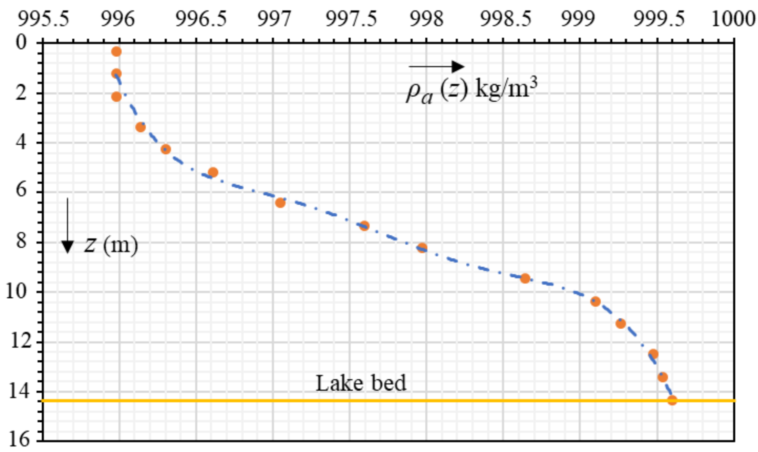

Figure 5: Vertical profile of ambient water density in the reservoir used for the numerical example

3) Choose a density current mean density *ρi*

To demonstrate the computation, we must specify the density of the density current at the downstream section, *ρ*(*t*) or *ρi* concept. In practice, *ρi* comes from inflow measurements, temperature and sediment concentration, or from the user model state.

For this numerical example, lets' take the following:

$$\rho_i = 998.9 \text{ kg/m}^3$$

This value is realistic for a moderately dense underflow/intrusion compared to warm epilimnetic water, but still within freshwater density ranges.

4) Compute the neutral depth *zi*

Recall that the neutral depth *zi* (Fig. 1) is the ambient depth where the current density equals the ambient density:

$$\rho_a(z_i) = \rho_i$$

Because

$$\rho_a(z)$$

is tabulated at discrete depths, compute *zi* by linear interpolation between the two depth points that bracket *ρi*.

Let

$$\left(z_1, \rho_1\right) \text{ and } \left(z_2, \rho_2\right)$$

be adjacent points with

$$\rho_1 \leq \rho_i \leq \rho_2$$

Then, the following can be written:

$$z_i = z_1 + \frac{\rho_i - \rho_1}{\rho_2 - \rho_1}\left(z_2 - z_1\right)$$

Since:

$$\rho_a(z_i) = \rho_i = 998.9 \text{ kg/m}^3$$

then Table 1 allows writing the following:

$$\rho_1 = 998.638384 < \rho_i = 998.9 < \rho_2 = 999.101575$$

Corresponding to the following (Table 1):

$z_1 = 9.4488 \text{ m}$, and $z_2 = 10.3632 \text{ m}$

Thus, the interpolation process gives what follows:

$$z_i = 9.4488 + \frac{998.9 - 998.638384}{999.101575 - 998.638384}\left(10.3632 - 9.4488\right)$$

The final result is as follows:

$$z_i = 9.96526442 \text{ m}$$

Thus, a current with $\rho i = 998.9$ kg/m^3 would be neutrally buoyant at about 10 m depth in this reservoir on that date.

5) Compute ψ using an explicit integral

The natural target depth is the neutral depth zi such as writing the following:

$$z* = z_i \tag{34}$$

and for simplicity, let us take the following entry at the surface:

$$z_e = 0 \tag{35}$$

Substituting Eqs. (34) and (35) into Eq. (31) yields the following, the dimensional work per unit mass available to adjust from the surface to the neutral depth:

$$\psi_i \equiv \psi(z_i) = \int_0^{z_i} g \frac{\rho_i - \rho_a(z)}{\rho_0} dz \tag{36}$$

Eq. (36) has units of energy per unit mass:

$$[\psi_i] = \mathrm{m}^2 / \mathrm{s}^2 = \mathrm{J/kg}$$

Let's define a dimensionless version of *ψi*. To do that, we must choose a reference value of the integrand and a reference vertical distance.

Define the following surface ambient density:

$$\rho_{a0} \equiv \rho_a(0) \tag{37}$$

Define the maximum (surface) density contrast between inflow and ambient as follows:

$$\Delta\rho_0 \equiv \rho_i - \rho_{a0} = \rho_i - \rho_a(0) \tag{38}$$

This

$$\Delta\rho_0$$

is the density anomaly the inflow "sees" at the surface.

Define a new reference reduced gravity having the same structure than Eq. (4). It reads as follows:

$$g_0^{'} \equiv g \frac{\Delta\rho_0}{\rho_0} \tag{39}$$

This is the reduced gravity that would obtain if the density contrast stayed equal to its surface value everywhere.

If the density contrast were constant and equal to

$$\Delta\rho_0$$

over the full depth interval 0 → *zi*, the reference "constant-contrast" work per unit mass would be as follows:

$$\psi_{\mathrm{ref}} = \int_0^{z_i} g \frac{\Delta\rho_0}{\rho_0} dz \tag{40}$$

Because

$$\Delta\rho_0$$

is constant with *z*, the integrand in Eq. (40) is constant and we can integrate exactly as follows:

$$\psi_{\text{ref}} = g\frac{\Delta\rho_0}{\rho_0}\int_0^{z_i} dz \tag{41}$$

Thus, the following can be written:

$$\psi_{\text{ref}} = g\frac{\Delta\rho_0}{\rho_0} z_i \tag{42}$$

According to Eq. (39), Eq. (42) can be written in the following reduced form:

$$\psi_{\text{ref}} = g_0' z_i \tag{43}$$

Eq. (43) is simply the "maximum" work that would get if stratification did not reduce the buoyancy advantage. It is a reference buoyancy work per unit mass that would occur if the buoyancy contrast stayed at its surface value.

Now let's define the dimensionless accessible-energy factor as follows:

$$\Psi = \frac{\psi_i}{\psi_{\text{ref}}} \tag{44}$$

Substituting Eqs. (36) and (42) into Eq. (44) yields the following:

$$\Psi = \frac{\psi_i}{\psi_{\text{ref}}} = \frac{\int_0^{z_i} g\frac{\rho_i - \rho_a(z)}{\rho_0} dz}{g\frac{\Delta\rho_0}{\rho_0} z_i} \tag{45}$$

After simplifying Eq. (45) becomes as follows:

$$\Psi = \frac{\int_0^{z_i} \left[\rho_i - \rho_a(z)\right] dz}{\Delta\rho_0 z_i} \tag{46}$$

Taking into account Eq. (28), Eq. (46) can be rewritten as follows:

$$\Psi = \frac{\int_0^{z_i} \left[\rho_i - \rho_a(z)\right] dz}{\left[\rho_i - \rho_a(0)\right] z_i} \tag{47}$$

Eq. (47) becomes as follows:

$$\Psi = \frac{1}{\left[\rho_i - \rho_a(0)\right] z_i} \int_0^{z_i} \left[\rho_i - \rho_a(z)\right] dz \tag{48}$$

Eq. (48) is the normalized form of your Eq. (31). The ratio Ψ in (48) is a dimensionless measure of stratification “how quickly the buoyancy advantage is consumed” as the parcel approaches the neutral depth.

If

$$\rho_a(z)$$

increases rapidly with depth (strong stratification), then

$$\rho_i - \rho_a(z)$$

drops quickly, the integral area is smaller, and Ψ is smaller.

If

$$\rho_a(z)$$

changes slowly (weak stratification), then

$$\rho_i - \rho_a(z)$$

stays large for longer, and Ψ is closer to 1.

Thus, Eq. (48) isolates the effect of the shape of *ρa*(*z*), independent of the overall scale. In addition, the integrand

$$\rho_i - \rho_a(z)$$

is the local density excess of the current relative to ambient at depth *z* (positive above *zi*, zero at *zi*. The denominator

$$\rho_i - \rho_a(0)$$

normalizes by the surface density contrast times depth, so ψ is dimensionless and typically $0 < \psi < 1$ for monotonic stratification.

Now, let's return to our numerical example

(6) Let's define the following, which is a part of Eq. (48):

$$I = \int_{0}^{z_i} \left[\rho_i - \rho_a(z) \right] dz \tag{49}$$

where

$\rho_i = 998.9 \text{ kg/m}^3$

From the real profile, we discrete depths z_k and corresponding ambient densities $\rho a(z_k)$, computed from the temperature data via the freshwater density polynomial.

For the integration we used the following:

(1) an added surface point which characteristics are reported in the first row of Table 1, i.e., same epilimnetic value as the shallowest measurement. Thus:

$z_0 = 0 \text{ m}, \ \rho_a(0) = 995.981068 \text{ kg/m}^3$

(2) all measured points down to

$z = 9.4488 \text{ m}$

meaning all measured depths shallower than the neutral depth, because all depths shallower than 9.4488 m are fully within the following buoyant range:

$\rho_i - \rho_a > 0$

The next measured depth, $z = 10.3632$ m reported in Table 1, is already denser than the inflow producing the following inequality:

$\rho_i - \rho_a < 0$

Therefore, the integral must: (1) include all measured points down to 9.4488 m, and (2) include one interpolated point at:

$z = z_i$, where $\rho_a = \rho_i$

Define the following integrand values:

$$f_k \equiv \rho_i - \rho_a(z_k) \tag{50}$$

The points used are reported in Table 2.

Table 2: Values of the integrand according to Eq. (50); ρ_i = 998.9 kg/m³

k	Depth z_k (m)	$\rho_a\left(z_k\right)$ kg/m³	$f_k \equiv \rho_i - \rho_a\left(z_k\right)$
0	0.0000	995.981068	2.918932297
1	0.3048	995.981068	2.918932297
2	1.2192	995.981068	2.918932297
3	2.1336	995.981068	2.918932297
4	3.3528	996.14193	2.758069896
5	4.2672	996.300021	2.599978717
6	5.1816	996.607783	2.292217196
7	6.4008	997.047958	1.852042492
8	7.3152	997.593557	1.306443092
9	8.2296	997.970683	0.929316886
10	9.4488	998.638384	0.261615717
11	z_i = 9.96526442	998.900000	0.000000000

Recall the following:

$$\rho_a(z_i) = \rho_i$$

This allows writing what follows, in accordance with Eq. (50):

$$f\left(z_i\right) = \rho_i - \rho_a\left(z_i\right) = 0 \quad (51)$$

We numerically integrate Eq. (49), using the trapezoidal rule, over each following interval:

$$\left[z_k \, , \, z_{k+1}\right] \quad (52)$$

Thus, one may write what follows:

$$I \approx \sum_{k=0}^{10} \frac{f_k + f_{k+1}}{2} \left(z_{k+1} - z_k\right) \quad (53)$$

Define the following each segment contribution:

$$\Delta I_k = \frac{f_k + f_{k+1}}{2} \left(z_{k+1} - z_k\right) \quad (54)$$

Now, proceed segment-by-segment calculations resulting in the values reported in Table.3.

Table 3: Values of ΔI_k according to Eq. (54)

k	Depth z_k (m)	f_k (Table 2)	f_{k+1}	$z_{k+1} - z_k$	ΔI_k Eq. (54)
0	0.0000	2.918932297	2.9189323	0.3048	0.889690564
1	0.3048	2.918932297	2.9189323	0.9144	2.669071692
2	1.2192	2.918932297	2.9189323	0.9144	2.669071692
3	2.1336	2.918932297	2.7580699	1.2192	3.460700536
4	3.3528	2.758069896	2.59997872	0.9144	2.449699826
5	4.2672	2.599978717	2.2922172	0.9144	2.236711972
6	5.1816	2.292217196	1.85204249	1.2192	2.526340706
7	6.4008	1.852042492	1.30644309	0.9144	1.444059609
8	7.3152	1.306443092	0.92931689	0.9144	1.022189462
9	8.2296	0.929316886	0.26161572	1.2192	0.725992515
10	9.4488	0.261615717	0	0.51646442	0.067557605

(7) Sum all contributions according to Eqs. (53) and (54); the following final result is obtained:

$$I \approx \sum_{k=0}^{10} \Delta I_k = 20.16108618 \qquad (55)$$

Recall the following values to insert into Eq. (48):

$\rho_a(0) = 995.981068 \text{ kg/m}^3$ (surface density given by Table 1)

$\rho_i = 998.9 \text{ kg/m}^3$

$z_i = 9.96526442 \text{ m}$ (Table 2)

Thus, using Eq. (55) and the previous density values, Eq. (48) gives the following:

$$\Psi = \frac{1}{(998.9 - 995.981068) \times 9.96526442} \times 20.16108618$$

The final result is as follows:

$\Psi = 0.69310834$

A value of $\Psi \approx 0.69$ means that about 69% of the maximum possible buoyancy work is actually accessible to the density current, given the real vertical stratification.

Equivalently, only 31% of the theoretical buoyancy potential is “lost” due to the rapid reduction of density contrast with depth. This is a highly non-trivial result.

On addition, $\Psi \approx 0.69$ is not a weak value. In strongly stratified lakes or reservoirs, it is very common that: (1) the density contrast collapses rapidly within the upper few meters, (2) Ψ takes values closer to 0.2-0.4, and (3) most of the potential buoyancy advantage is consumed very early.

By contrast, $\Psi \approx 0.69$ indicates that: (1) the density contrast between inflow and ambient persists over a substantial fraction of the descent, (2) the stratification is moderate rather than extremely sharp, and (3) the inflow retains buoyancy advantage deep into the water column.

In practical terms, $\Psi \approx 0.69$ corresponds to a stratification that is permissive to density-current persistence. Moreover, the computed Ψ is consistent with the observed density profile: (1) Near-surface densities are much lighter than the inflow, (2) Density increases progressively with depth, (3) The neutral depth is reached only after a significant descent, (4) Available buoyancy power is nearly 70% of its theoretical maximum, and (5) Entrainment and drag losses are more easily “paid”.

A useful interpretive scale is reported in Table 4.

Table 4: Physical interpretation of Ψ with respect to its value (Authors’ own values)

Ψ value	Interpretation
$\Psi < 0.3$	Strongly limiting stratification; rapid loss of buoyancy
$0.3 < \Psi < 0.5$	Moderately restrictive stratification
$0.5 < \Psi < 0.8$	Energetically favourable stratification
$\Psi \approx 1$	Nearly uniform ambient density

Thus, the computed value $\Psi \approx 0.69$ lies firmly in the energetically favourable regime. It places the numerical example in a physically realistic, informative regime, ideal for demonstrating the value of the advocated APEB framework.

As a concluding sentence, one may write the following:

The computed dimensionless energy factor $\Psi \approx 0.69$ indicates that nearly 70% of the maximum buoyancy work theoretically available at the surface remains accessible over the descent to the neutral depth, revealing a stratification that is energetically favourable to density-current persistence rather than strongly limiting.

Novel contribution: Dissipation demand and persistence criterion

Mixing/entrainment power demand

Entrainment introduces turbulence and mixing losses. We model the minimum power required to sustain entrainment as scaling with the entrainment volumetric rate per unit width expressed by Eq. (5) recalled as follows:

$$\frac{d}{dx}(q) = w_e b \tag{5}$$

and kinetic energy per unit mass $\sim u^2$:

$$P_m = C_m \rho_0 (w_e b) u^2 \tag{56}$$

Substituting Eq. (6) into Eq. (56) yields the following:

$$P_m = C_m \rho_0 (Eub) u^2 \tag{57}$$

Thus, after rearrangement, Eq. (57) becomes as follows:

$$P_m = C_m \rho_0 E b u^3 \tag{58}$$

It worth noting that in classical density-current theory, dissipation is often represented implicitly through: (1) empirical entrainment coefficients, (2) bulk drag terms in the momentum balance, (3) or assumed steady-state Froude conditions.

However, these approaches do not explicitly account for the energetic cost of maintaining entrainment itself. Entrainment is not merely a geometric process; it is a turbulent mixing process that requires continuous energy input. In the present study, the parameter *Pm*, expressed by Eq. (58), is introduced to explicitly represent this energetic cost.

Pm is defined as the rate at which mechanical energy must be supplied by the density current to sustain turbulent entrainment and internal mixing with the ambient fluid. More precisely, *Pm* is the power required to continuously mix entrained ambient water into the density current so that the current remains dynamically coherent. It has units of power (Watts).

From a physical interpretation point view, *Pm* represents the energy required to: generate and sustain turbulent eddies at the current-ambient interface, overcome stratification locally during entrainment, accelerate newly entrained ambient fluid from rest (or ambient velocity) to the velocity of the current, and homogenize the entrained fluid with the current interior.

This energy is not associated with bed friction or form drag, those are treated separately by P_f and P_i. Instead, *Pm* is the internal mixing power.

As indicated above, it should be noted that entrainment has an energetic cost. Indeed, when ambient fluid is entrained: (1) It initially has different momentum than the current, (2) It must be accelerated to the current velocity, and (3) It must be mixed against a stabilizing density gradient, especially in stratified reservoirs.

Each of these steps requires energy, which ultimately comes from the buoyancy-driven motion of the density current itself.

The introduction of *Pm* is a conceptual advance, not because mixing is new, but because: (1) Classical models assume entrainment occurs once a current exists, (2) The present framework asks whether the current has enough energy to afford entrainment.
In this sense, *Pm* is the energetic gatekeeper of density-current persistence.

Pm is modelled by Eq. (56) which follows directly from physical considerations:

(1) the product

$$\left(w_e b\right)$$

is the volumetric entrainment rate per unit length ($m^3\ s^{-1}\ m^{-1}$),

(2) The quantity

$$\rho_0 \left(w_e b\right)$$

is the mass entrainment rate per unit length,

(3) C_m

is an efficiency coefficient capturing turbulent mixing losses and anisotropy.

(4) u^2

is the characteristic kinetic energy per unit mass required to bring entrained fluid up to current velocity.

Frictional and interfacial drag demand

Bed and interfacial drag produce dissipation respectively as follows:

$$P_f = \tau_b u b \tag{59}$$

$$P_i = \tau_i u b \tag{60}$$

with the following quadratic stress closures

$$\tau_b = C_d \rho_0 u^2 \tag{61}$$

$$\tau_i = C_i \rho_0 u^2 \tag{62}$$

where

τ_b

is the shear stress exerted by the bed on the flowing density current,
while

τ_i

is the shear stress acting at the interface between the density current and the ambient fluid. It represents the momentum flux per unit area transferred from the current to the ambient due to turbulence.

Using Eqs. (61) and (62), Eqs. (59) and (60) become respectively as, follows:

$$P_f = C_d \rho_0 b u^3 \tag{63}$$

$$P_i = C_i \rho_0 b u^3 \tag{64}$$

P_f represents the rate of mechanical energy dissipation due to bed friction exerted by the solid boundary (lake or reservoir bed) on the density current. In physical terms, it is the power lost by the density current to the bed as a result of viscous and turbulent shear stresses acting at the flow–bed interface. This loss occurs because: (1) the current must continuously do work to overcome resistance imposed by the rough, immobile bed, and (2) near-bed turbulence and shear convert organized kinetic energy of the current into heat.

Thus, it is an irreversible sink of mechanical energy; an energy dissipated at the solid bed.

P_i represents the rate of mechanical energy dissipation due to interfacial drag between the density current and the overlying ambient fluid. In physical terms, it is the power lost by the density current as a consequence of turbulent shear and momentum exchange across the current-ambient interface. This dissipation arises because: (1) the density current and the ambient fluid generally move at different velocities, (2) velocity shear at their interface generates turbulence, (3) turbulent eddies transport momentum across the interface, and (4) kinetic energy of the current is irreversibly converted into heat and background mixing.

Thus, it is an irreversible sink of mechanical energy, distinct from bed friction and internal mixing.

Total demand

Total demand is expressed as follows:

$$P_D = P_m + P_f + P_i \tag{65}$$

Substituting Eqs. (58), (63) and (64) into Eq. (65) yields the following final result:

$$P_D = \rho_0 b u^3 \left(E C_m + C_d + C_i \right) \tag{66}$$

Table 5 below show the distinction between *Pm* and other dissipation terms, as it is important to distinguish *Pm* from other power sinks.

Table 5: Distinction between dissipation terms used in this study

Term	Equation	Physical meaning
P_m	(56)	Power required for internal turbulent mixing and entrainment
P_f	(63)	Power dissipated by bed friction
P_i	(64)	Power dissipated by interfacial shear with ambient flow

The APEB persistence criterion (new predictive condition)

We propose that a coherent density current persists only if accessible buoyancy power exceeds the required dissipation demand; this can be translated as follows:

$$\Phi > P_D = P_m + P_f + P_i \tag{67}$$

The core novelty of the framework is the above Accessible Potential Energy Budget (APEB) condition.

Within this inequality, it can be written the following:

Φ is the maximum buoyancy power available from stratification, *Pm* is often the dominant sink in strongly stratified reservoirs, if Φ < *Pm*, the current cannot sustain entrainment and collapses into diffuse mixing.

Thus:

Pm controls whether entrainment is energetically affordable.

Moreover, if *Pm* were omitted, the following can occur: (1) the model would predict density-current persistence even when buoyancy input is insufficient to sustain mixing, (2) and the theory would reduce to a force-balance framework and lose predictive power regarding collapse with respect to persistence.

Including *Pm* ensures the model respects the first law of thermodynamics: energy must come from somewhere.

As a one-sentence summary, one may write the following:

The parameter *Pm* represents the mechanical power required to sustain turbulent entrainment and internal mixing within the density current, accounting explicitly for the energetic cost of incorporating ambient fluid into the current and thereby governing whether a coherent density current can persist.

Using Eq. (31a) and the inequality (67) yields the following final result:

$$Q_0 \int_{z_e}^{z_i} g \frac{\rho_i - \rho_a(z)}{\rho_0} dz > b u^3 \left(E C_m + C_d + C_i \right) \tag{68}$$

Novelty statement: the inequality (67) and Eq. (68) are the key new result: a persistence criterion that links stratification

$\rho_a(z)$, the entry z_e, inflow density ρ_i, and flow geometry/velocity (b,u) in a single, testable inequality.

Scaling: nondimensional "Reservoir Density-Current Number"

Define a new nondimensional number

$\mathbb{R}$

as the following ratio:

$$\mathbb{R} = \frac{\Phi}{\rho_0 b u^3} \tag{69}$$

$\mathbb{R}$ measures how much buoyancy-power the stratification makes available relative to the power needed to maintain the current at speed *u* and width *b*.

Recall that Φ is governed by Eq. (31b). Thus, the following can be written:

$$\mathbb{R} = \frac{\Phi}{\rho_0 b u^3} = \frac{\rho_0 Q_0 \psi_i}{\rho_0 b u^3} \tag{70}$$

This can be simply written as follows:

$$\mathbb{R} = \frac{Q_0 \psi_i}{b u^3} \tag{71}$$

From Eq. (70), the following can be written:

$$\Phi = \rho_0 b u^3 \mathbb{R} \tag{70a}$$

Let's define the composite loss coefficient as follows:

$$\Lambda \equiv E C_m + C_d + C_i \tag{72}$$

So, Λ is an effective "loss factor" collecting entrainment and drag.

Taking into account Eq. (66), the inequality expressed by Eq. (67), and Eq. (70a), yields the following final result:

$$\mathbb{R} > \Lambda \tag{73}$$

From a physical point view, the inequality (73) tells us the following. A reservoir density current can persist as a coherent underflow/interflow only if the buoyancy-work "budget" available from stratification is larger than the mechanical "cost" required to (1) overcome frictional drags, and (2) maintain turbulence/entrainment/mixing.

CONCLUSION

Density currents in stratified dam reservoirs are central to sediment transport, water quality evolution, and reservoir operational performance. While decades of research have established robust descriptions of how such currents evolve once formed, classical gravity-current theory has largely taken the existence and persistence of density currents as given, relying on force balances, geometric intrusion criteria, or empirical entrainment closures. The energetic feasibility of sustaining a density current within a stratified environment has remained an open and practically important question.

In this study, we introduced an Accessible Potential Energy Budget (APEB) framework to explicitly address this gap. The central idea of the framework is that ambient stratification is not merely a geometric constraint determining plunge or intrusion depth, but an energetic resource that governs whether a density current can exist and persist as a coherent flow. By integrating the ambient density profile along the vertical buoyancy-adjustment pathway of the inflow, we defined a physically interpretable quantity: the accessible buoyancy work per unit mass, which directly links measured reservoir stratification to an energy scale. When multiplied by the inflow mass flux, this quantity yields a buoyancy-derived power input that represents the maximum mechanical energy that stratification can supply to a density current.

A key contribution of this work is the explicit decomposition of the dissipation demand faced by a density current into three distinct components: (1) internal turbulent mixing and entrainment power, (2) bed-friction dissipation, and (3) interfacial drag with the ambient fluid. Unlike classical formulations in which entrainment is treated implicitly through empirical coefficients, the present approach recognizes entrainment as an energetically costly process that must be "paid for" by buoyancy-derived energy. The resulting APEB persistence criterion states that a coherent density current can exist only if the accessible buoyancy power exceeds the total dissipation demand. This inequality constitutes a new predictive gatekeeper that distinguishes between persistent density currents and flows that rapidly collapse into diffuse mixing.

The worked numerical example based on real reservoir stratification data demonstrates the practical applicability of the framework. The example shows that the shape of the ambient density profile, encapsulated in a dimensionless stratification factor, exerts a dominant control on the accessible energy. Even for identical inflow density and discharge, different stratification profiles can yield markedly different energetic outcomes, explaining why some reservoirs exhibit long-runout underflows or interflows while others do not. The computed stratification factor illustrates how much of the theoretical buoyancy potential is actually accessible, providing a quantitative diagnostic of stratification favourability that cannot be obtained from classical Froude-based or force-balance approaches.

Conceptually, the APEB framework extends classical density-current theory without redefining its foundational governing equations. Instead, it adds a missing energetic dimension that unifies underflows and interflows within a single physical perspective and clarifies the role of neutral depth as both a stratification constraint and an energetic boundary. The framework transforms stratification from a descriptive backdrop into an active participant in the dynamics, capable of enabling or suppressing density-current persistence.

From a practical standpoint, the proposed approach offers new tools for reservoir management. Because the accessible potential energy can be computed directly from field-measured temperature or density profiles, the framework provides a means to anticipate density-current behaviour under varying stratification conditions, seasons, or operational scenarios. This has direct implications for sediment management strategies, the timing of low-level withdrawals or venting operations, and the assessment of water-quality risks associated with hypolimnetic transport.

Finally, the APEB framework opens several avenues for future research. Extensions may include accounting for the evolution of inflow density during vertical adjustment and downstream propagation, coupling the energy budget with time-dependent stratification, and integrating sediment settling or thermal exchanges into the energetic balance. More broadly, the approach provides a bridge between reservoir hydraulics and modern stratified-flow energetics, suggesting that energy-based diagnostics can play a central role in advancing predictive capability for density-driven flows in natural and engineered water bodies.

In short, this study demonstrates that the persistence of density currents in stratified dam reservoirs is fundamentally an energetic question, not solely a geometric or kinematic one. By explicitly quantifying the balance between buoyancy-derived energy and dissipation demand, the Accessible Potential Energy Budget framework provides a new, physically grounded criterion for density-current formation and persistence, thereby extending classical theory and offering practical insight for reservoir hydraulics and management.
Classical layer-averaged equations describe how a gravity current evolves assuming it forms and persists. The APEB framework adds predictive capability by addressing: (1) Formation/persistence with respect to collapse: whether the current remains coherent or rapidly mixes out, (2) Sensitivity to stratification shape: via the integral in Eq. (31a), (3) Unified regime interpretation: underflows and interflows are treated consistently through the role of z_i and the accessible energy.

One can also derive the alternative version where the parcel density evolves due to entrainment during descent; then ρ_i in Eq. (31) is replaced by a downstream-evolving $\rho_t(x)$, and ψ becomes path-dependent. That would be a natural "next-level" refinement of the APEB framework. This is what the authors will develop in a future study.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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